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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

COMBINING GEODETIC SURVEY METHODS WITH CADASTRAL SURVEYS

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Synopsis

The purpose of this paper is to present the field and office technique developed on a large reservoir boundary survey, in which the economic consideration of the integral parts of such a survey has resulted in a more feasible and less costly treatment of land acquisition survey practise.

Introduction

The Grand Coulee Dam is situated on the Columbia River in northeastern Washington, 153 miles down stream from the point where the river enters the State. Four tributary rivers of importance—the San Poil, Spokane, Colville, and Kettle rivers—flow into the Columbia River at distances of 19, 47, 107, and 114 miles, respectively, up stream from the dam site. Numerous other streams of minor importance are tributary to the Columbia throughout the backwater area, some entering in narrow, precipitous canyons, and others across comparatively wide distributary delta plains. In all, a shore line of about 500 miles will be created by the lake behind this structure, the largest man-made structure in the history of the world. The deep Columbia Valley is characterized throughout as consisting of softly molded rolling hills and valleys, and lends itself most favorably to the development of a strong network of triangulation for co-ordination and control.

Summarized, the purpose of the boundary survey was: (*a*) To determine the extent and area of all real property situated and lying below a general taking line, or flow line, the elevation of which was fixed at 1 310 ft above mean sea level, and to fix and define the boundary lines of the area by executing a traverse of unbroken continuity lying along, or just above, the 1 310 contour; (*b*) to control the flow-line traverse properly with a network of triangulation of such accuracy as to hold the closing errors of the traverse within the prescribed limits; (*c*) to recover or properly re-establish the original survey corners and other cadastral points established by the General Land Office and reference

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **January 15, 1940**.

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them to the rectangular co-ordinate system; (*d*) to furnish the Legal Division of the United States Bureau of Reclamation with analyses of chains of title, in the cases of parcels having obscure or inaccurate descriptions in previous conveyances, and finally to furnish the Legal Division with properly closed descriptions of all parcels of real property involved; and (*e*) to monument the network of control triangulation permanently and connect this network to the adjacent existing first-order triangulation of the U. S. Coast and Geodetic Survey, thus referencing the entire system of co-ordinates to the Federal network on the 1927 North American Datum.

Methods of making control surveys, within themselves, are not new to the profession; nor are the many accoutrements of retracement and flow-line surveys. The combination of the two into a single, simultaneous operation encompassing the use of instruments ranging from the simple hand compass to the 12-in. direction theodolite, and of distance determination from the woodsman's pacing to invar taping, introduces a new procedure, and is distinctly a problem in surveying economics.

Specifications

Early in the work, it was decided that all project co-ordination should be placed on the horizontal and vertical data commonly accepted by the twenty-three members of the Federal Board of Surveys and Maps, of which organization the U. S. Bureau of Reclamation is a member. Thus all surveys would be referenced permanently to the National network and would provide not only a lasting usefulness of all Bureau surveys to Federal, State, and local agencies, but likewise would be of advantage to the Bureau in any future work performed by other agencies in this locality. Consequently, all horizontal control was to be referenced to the 1927 North American Datum, which has as its basis the Clarke Spheroid of 1866. Vertical control was to be referenced to the 1929 level net adjustment of the North American continent.

The requirement was established that all work be "check-position"; that is, that all position computations in the field would be made by at least two independent mathematical solutions, and that by a comparison of the two routes of computation a ratio check of the actual field error would result. This ratio check was set by specifications to be not more than one part in 5 000 for all reservoir co-ordination. Both the horizontal and vertical geodetic control were to be established by triangulation and precise leveling methods, adhering to the specifications and to office and field procedure of the U. S. Coast and Geodetic Survey.

Co-Ordinates

In 1933–34 the Bureau of Reclamation established Triangulation Station Alpha at Latitude 47° 58′ 00″.844 and Longitude 118° 58′ 29″.827 as the origin of a rectangular co-ordinate system for the Columbia Basin Project. The geographic position was determined by first-order and second-order triangulation and adjusted by the direction method to the adjacent Umatilla-49th Parallel arc of the U. S. Coast and Geodetic Survey. Rectangular co-ordinates, raised to the datum plane 1 310 ft above sea level, were then computed.[2] This

[2] Computed from formulas in *Special Publication No. 71*, U. S. Coast and Geodetic Survey.

grid was designated the Grand Coulee Grid Datum (GCGD), and its use was extended to a point approximately 30 miles east, and at nearly the same latitude, as Station Alpha. The practise was, after adjustment, to convert the geographic positions of the main-scheme second-order stations to grid co-ordinates; then from grid distances and bearings derived from these co-ordinates, to compute grid co-ordinates of all intersection and resection third-order stations within the figures. The scale error, increasing with the departure from the origin, had enlarged to the point at which, if use of the Grand Coulee Grid Datum was to continue, it would be necessary to compute all third-order points first on true geodetic datum and then convert individually to the rectangular grid. The reason for this proposed conversion was to smooth out, proportionately, all grid discrepancies because, with departure from the origin, the grid scale was too long with respect to an east and west direction, too short in a north and south direction, and lines perpendicular on the spheroid were several seconds askew from normal on the grid. Thus, although the accuracy of the plane co-ordinates was well within one part in 25 000, or first-order, at the mouth of the Spokane River, the discrepancy would approximate one part in 3 500 at the point where the Columbia River enters the State.

Toward the close of 1935 the Lambert Conformal projection tables—one covering the north half, and another the south half of the State—were published for the State of Washington. The wide-spread adoption of similar systems by other States, particularly those in the East, quickly led to the decision to place all future Bureau co-ordination over the reservoir site on the North Washington System.

Retracement

Early in 1934 two retracement parties were organized; they began at the dam site and proceeded up stream in search of the General Land Office corners, some of which had been established by contract surveyors more than fifty years before. Each party, comprising seven men, was assigned one bank of the river, and pursued its search by the stadia-transit traverse method. Where corners or their accessories were lost a random hub was set, and the party subsequently returned to re-establish the corner by the proportionate measurement method.

The following year (1935) a part of the retracement work was assigned to the triangulation party, and the time-honored "timber-cruiser" method was inaugurated on the project. Two seasoned woodsmen, well trained and experienced in finding obscure bearing trees and bits of evidence, were employed. The woodsmen, starting from a known corner, and armed only with photostatic copies of the General Land Office original field notes, a foresters' staff compass or a hand compass, paced off the required distance in a cardinal direction, and recovered or marked for re-establishment the corner sought for.

Upon arrival at the scene of a corner the retracement engineer first made the usual diligent search for evidence of the corner monument or the corner accessories, as described in the original field notes. If found, or restored either from the accessories or upon reliable parol (word of mouth) evidence obtained from some local landowner, the full description was entered on a Cadastral

Retracement Record form (see Fig. 1) and the corner designated as a Type "A" recovery. Often a diligent search of old records in Court houses or abstract offices would reward the efforts of the engineer by disclosing a reliable tie from some near-by identifiable object to the now lost corner. In a few cases Court adjudications locating the positions of disputed corners would form the basis of the recovery record.

Failure to recover a Type A, or original corner, would lead to interviewing the adjoining landowners for support of established fence corners. If they were found to have been established beyond the statutory length of time required for the enjoyment of peaceful occupation by the abutting owners, these points were designated as Type "B" corners, or legal corners of occupancy. Here again, all available parol evidence was entered on the Cadastral Retracement Record form, and the attest of the parties interviewed was obtained.

After all possibilities of typing a corner as "A" or "B" were exhausted, the re-establishment method by proportionate measurement, or Type "C," was resorted to, and the methods prescribed by the General Land Office manual were followed rigidly.

Signaling

Accompanying each retracement engineer was a signalman who, in addition to lending assistance in the recovery of the cadastral points, would determine which of three general methods was most applicable for use in co-ordinating the point. The man performing this detail of the work was an important adjunct to the organization; his ability and developed judgment reflected a marked influence on the final cost of the newly developed procedure.

Points were co-ordinated either as: (*a*) Full intersection stations, (*b*) as resection stations, by the Pothenot method,[3] or (*c*) as traverse stations.

The first method was subject to a number of variations in procedure. If a point was visible from three triangulation stations and otherwise possessed the condition of location essential to a strong "fix," it was "signaled" by placing a 4 in. by 4 in. signal pole directly over the corner-stone itself and the method of co-ordination in this case was designated "Intersection station, central." More often, however, a 4 in. by 4 in. witness post, set from 6 in. to 20 ft away from the point, was used as the signal; the distance was measured carefully and the eccentric azimuth was read from a staff compass. This method was designated "Intersection station, point near." This latter method, although introducing one additional step in the computation work, obviated the necessity of plumbing and guying a signal pole over the stone monument, which is objectionable chiefly because livestock would often push over such a signal. On the other hand, a 4-ft post, set 18 in. in the ground, was secure from this danger. Where signal posts were set beyond the 20-ft limit from the corners, the designation "Intersection station, transit eccentric" was used, and the distance and direction were obtained by the signalman with a metric-foot tape and a lightweight transit. Often the densities of Indian allotment and cemetery corners, traverse control points, and adjacent section corners, would permit observation

[3] "Criteri e procedimenti per la formazione della mappe catastali con la fotogrammetria aerea," by Michele Tucci, R. Istituto Superiore di Ingegneria, Milano, 1935, pp. 16 and 18.

UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

CADASTRAL RETRACEMENT RECORD

INDEX GC-1
S.B.C. 5003 A
G.L.O. 64
F.B. ______
Pg. ______

Corner 1/4 S. 33-34 Twp. 28 N. Rge. 28 E.W.M.

From original G.L.O. field notes of 1881 (year)

Corner: 1 qt. charcoal—12" deep.

Witness Marks: stake in north pit mkd. 1/4 S.; Pits; ______; Mound of earth

Evidence found Dec. 13 (date), 1937

No original evidence.
Position of original corner sustained by Douglas County Engrs. field notes, Road No. 90, 1889, and attested by adjacent owners.

Type "A" Original G.L.O. found ______ or restored ______.

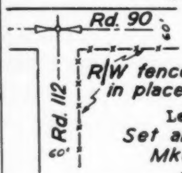
Original corner stone ______
Original corner post ______
Reset from ______ BT's ______ and marked with ______
Reset from ______ WM's ______ and marked with ______
By parol evidence of M ______ and M ______
address: ______ ______
Taken by: ______ Retracement Engineer Date ______, 19__

Do not write in margin

Type "B" Legal corner of occupancy ✓ Description: ℄ intersection of E.&S. rds.

Date established, approximately, ______, 1889

Attest of adjacent concurring owners:



M [illegible], owner in sec. 33
M [illegible], owner in sec. 34-33
M [illegible], owner in sec. 34
M ______, owner in sec. ______

Legal ~~fence~~ roads (s) constructed: east ✓ about year 1889; south ✓ about year 1889; west ______ about year ______; north ______ about year ______

Set an 8"x10"x12" basalt stone Mkd. 1/4 S. on north face —H.C.L. 12-13-37

Type "C" Reestablished by U.S.B.R. retracement survey, 19__, by single proportionate ______, double proportionate ______ method.

Corner marked with ______ (post) (stone) (pipe)

Witness marks: ______

By: H. C. Lake
Retracement Engineer

Approved: [illegible]

FIG. 1.—CADASTRAL RETRACEMENT RECORD FORM

upon from eight to ten "transit-eccentrics" from a single setup on an intersection station.

Necessity for the second method, that of the resection, or Pothenot method, arose chiefly from three situations: (*a*) That in which the second-order observing party failed to complete each of the prescribed "cuts" to a signal, either because the signal was disturbed or because of poor light and consequent poor conditions of visibility; (*b*) that in which, because of a corner poorly located for the requirements of a strong intersection, use of a three-point-fix observation from some advantageous point near the corner would enable the observer to select and make use of other non-occupied stations, as spires, cupolas, water-towers, etc., for a strong "fix"; or (*c*) that in which, subsequent to second-order observation over an area, it was decided to co-ordinate additional corners. In deciding upon the use of this method, the signalman left the choice of the "point near" and the objects to be observed upon, to the judgment of the three-point observer. Only a flag or banner was left at the corner to assist the observing party in finding the corner.

The third, or traverse method, was necessitated where dense stands of timber prohibited the use of triangulation methods. Three such traverses were used, two along the Columbia River near the town of Lincoln, Wash. (see Fig. 2), and one on the Spokane River near Little Falls, Wash., where "fixes" at each end of the traverses made it necessary to tape only in one direction. A fourth invar traverse was taped along an old railway road-bed in the Hawk Creek Canyon, and since a "fix" was impossible at the upper end, the traverse was double-taped to meet the requirement of check position. Corners and control points adjacent to these precise traverses were co-ordinated either by metric-foot taping, or by making them intermediate angle points in transit and steel tape loop traverses of third-order accuracy originating and ending at points along the precise traverses.

Triangulation

First-Order.—First-order triangulation surveys comprised a chain of 25-mile quadrilateral figures extending from the Grand Coulee dam site east and north over the reservoir area to the International Boundary, and south from the dam site over the irrigation balancing reservoir area in the Grand Coulee to Coulee City, Wash., tying in to existing stations of the U. S. Coast and Geodetic Survey at each end, and to stations of the Geodetic Survey of Canada north of the Boundary. The network thus established extended to the high mountain peaks adjacent to the reservoir areas and established solitary control stations of first-order accuracy at about 25-mile intervals slightly above the reservoir flow lines. At the same time similar control stations were fixed in position at the heads of tributary reservoirs in the San Poil, Spokane, and Kettle rivers.

Besides the two 50-m invar tapes and stretching apparatus for measuring base lines, the Bureau obtained two instruments for first-order observations from the Coast and Geodetic Survey, a 12-in. direction theodolite, and a 7-in. repeating vertical circle.

Day-time observations of double zenith distances were made upon distant helioscope signal lights, using the vertical-circle instrument. Angles were

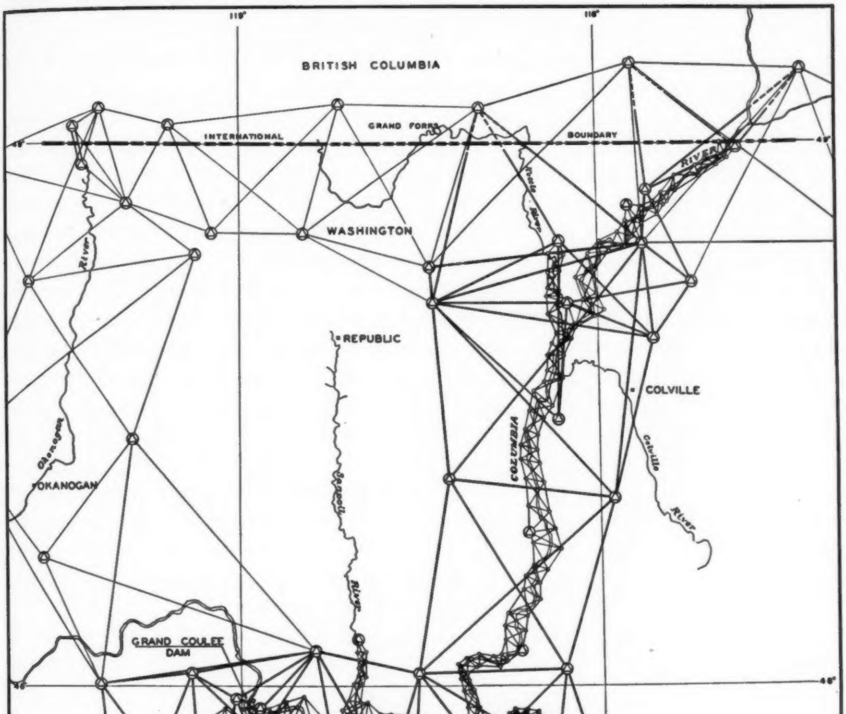


Fig. 2.—Geodetic Control Surveys—Columbia Basin Project

read on each of four verniers, direct to 10″. These latter observations formed the basis of the trigonometric leveling performed over the project.

In the Summer of 1934 a camp was established on the Columbia River 60 miles down stream from the International Border. Eight men were given a rigid course of training in the many duties and responsibilities of lightkeepers. Next, their adaptability for this rigorous and exacting work of back-packing, camping, and flashing code messages with helioscopes and electric signal lights, was tested by sending them out alone for two or three nights of work on a lone mountain peak. Those men found to be qualified were given regular assignments on reconnaissance work to locate all the stations of the first-order net. The actual observing work was next undertaken; 42 first-order stations were occupied, and the geographic positions of a total of 97 points determined. Among these were a number of fire-control towers, and their positions and azimuths between towers later were made available to the U. S. Forest Service, the U. S. Indian Field Service, and the State of Washington Division of Forestry. A number of aviation beacons were likewise "cut in," and their positions supplied to the U. S. Bureau of Air Commerce. Mapping operations of the U. S. Geological Survey were in progress near Kettle Falls during the seasons of 1935–1936, and positions of a number of their topographic control flags were supplied that Bureau. Still later, a part of the first-order net was used in controlling flights of aerial photographs taken by the Soil Conservation Service. First-order control for the project extended over an area of 3 038 sq miles.

Second-Order.—Between the control stations established by first-order work along the reservoir site, a secondary network, made up of quadrilateral figures averaging 1.75 miles in axial length, was monumented and observed. Lengths were expanded into the figures at the ends of each net from base lines measured with second-order accuracy, and, after making least-squares adjustment between the control stations, the latitudes and longitudes of the triangulation stations were converted to rectangular grid co-ordinates to simplify their use in conveying tracts. Spacing of first-order control stations along the flow lines permitted breaking up the second-order work into seven nets along the Columbia River. Three additional nets extended into the valleys of the San Poil, Spokane, and Kettle rivers. Division of the nets in this manner offered the advantage of having available, for the Right-of-way Division of the Bureau of Reclamation, the final "closed" plats and descriptions of any tract, within a few weeks after the second-order and third-order work in that net had been completed.

Measurements of angles between main stations, directions to cadastral signals and flow-line control points, and double-zenith-distance measurements for trigonometric leveling, were all observed simultaneously from the second-order stations; 7-in. theodolites reading to 10″, with single, opposite verniers, were used. Although this theodolite was of the "repeating" type, all pointings were made by the "direction" method, the high quality of the circle graduations permitting this variation. On the main network, 12 positions of the circle were read, with a residual of 5 sec from the mean specified as a maximum, whereas three positions were used in pointing the cadastral signals, and the same residual specified. Use of the direction method often permitted a station to

be completed in much less time than it could have been by the method of repetition, because of poor light or fog obscuring some of the signals during a part of the observing period. Likewise, the direction method obviated the necessity of making the consequent local adjustment of sum angles in the lists of direction.

The parties occupied 342 second-order stations and determined the geographic positions of 2 020 points. An example of the notes for seven of these stations is presented in Fig. 3. Second-order control for the project extended over 437 sq miles.

Third-Order.—By triangulation or traverse of third-order accuracy, or by some combination of the two, the positions of all cadastral points within, and adjacent to, the reservoir area were co-ordinated and referenced to the existing monumented stations. In most cases this was accomplished by measuring the angular directions to signals set at or near the retraced cadastral monuments or flow-line angle points; and the procedure, because of its closely related performance to the second-order work, has already been described herein. The few remaining corners which lay concealed from second-order stations were coordinated by three-point-fix observations made from an open spot near the concealed monument. The practical aspects of this, the Pothenot method, or Italian "metodo di Pothenot," [3] are already rather well known to professional photogrammetrists, and use of this method represents the refinement necessary to its adaptation to cadastral surveying.

Instructions to the "three-point" observing party emphasized largely the necessity of selecting stations carefully for a strong resection, and further pointed out the dangers of a "fix" lying on or near the arc of the great circle, such a fix (commonly termed a "swinger") being indeterminate. Treatment of the resection problem may be found in any good manual on plane-table surveying, and the only further mention of the procedure in this paper is of the development of an arbitrary index to the strength of any given three-point-fix, which will be found with the description of the computing form in use on this project.

Rather frequently a landowner would appeal to the Right-of-way Division to complete appraisal and survey of his tract in advance of other work in that vicinity to forestall some pending litigation, and thus save both himself and the Government considerable delay and expense by the speedy consummation of the property transaction. In that case the three-point party would coordinate the adjacent corners and traverse control points quickly, and the necessary plat would be forthcoming within three or four days.

The use of a standardized metric-foot tape was found most applicable for co-ordinating points within 100 ft of the control station. For points lying beyond 100 ft and up to 500 ft, or for taping loop traverses, a standardized 300-ft or 500-ft steel tape was used. A standardization site was laid out with the invar tapes, and a calibration card was compiled for each of the two lengths of tapes. These cards were given to the chainmen for field use. Mounted on cardboard, they furnished a rapid means of determining the proper plus or minus correction for various conditions of temperature and catenary. For

UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

GEOGRAPHIC POSITIONS

58

Accession No. of Computation: ______

Locality Columbia Basin Project - Grand Coulee Dam *Datum* North American 1927 *State* Washington

STATION		LATITUDE AND LONGITUDE ° ′ ″	SECONDS IN METERS	AZIMUTH ° ′ ″	BACK AZIMUTH ° ′ ″	TO STATION	DISTANCE LOGARITHM (METERS)	METERS	FEET
TIP, 1936	d.m.	48 48 04.3960		42 42 59.40	222 40 47.25	BOSSBURG	3.723 3204	5,288.352	17,350.20
(Y 669,986.57 ft.)		117 57 36.5908		79 48 22.49	259 45 10.65	DILLY	3.723 2693	5,287.730	17,348.16
(X 2,692,357.07 ft.)									
LANE, 1936	d.m.	48 49 11.7766		11 12 01.33	191 11 17.78	BOSSBURG	3.784 1731	6,083.775	19,959.85
(Y 676,518.08 ft.)		117 59 34.3980		48 50 34.39	222 48 51.17	DILLY	3.614 6211	4,117.381	13,508.44
(X 2,684,223.02 ft.)				310 52 42.77	130 54 11.42	TIP	3.502 3708	3,179.588	10,431.70
DENNY, 1936	d.m.	48 50 07.0080		12 32 31.82	192 32 00.74	TIP	3.588 8416	3,880.088	12,729.92
(Y 682,507.78 ft.)		117 56 55.3012		62 16 55.90	242 14 56.14	LANE	3.564 2116	3,666.161	12,028.06
(X 2,694,653.89 ft.)									
RYAN, 1936	d.m.	48 47 59.5422		95 13 20.24	275 12 19.66	TIP	3.217 4732	1,649.959	5,413.24
(Y 669,696.99 ft.)		117 56 16.0734		118 53 39.89	298 51 10.64	LANE	3.664 7303	4,620.939	15,160.53
(X 2,697,762.63 ft.)				168 30 56.79	348 30 27.26	DENNY	3.604 0095	4,017.996	13,182.38
O'TOOLE, (C. & G.S.), 1925		48 48 32.364		74 45 30.01	254 19 15.51	COPPER BUTTE	4.647 1533	44,376.53	145,592.0
(Y 673,524.06 ft.)	d.m.	117 52 58.201		75 55 29.94	255 53 01.04	RYAN	3.619 4121	4,163.055	13,658.29
(X 2,710,873.98 ft.)				121 10 38.99	301 07 40.52	DENNY	3.752 1604	5,651.457	18,541.49
r. 1934, r. 1936				305 58 40.07	126 05 04.42	ROGERS, 1933	4.110 6516	12,901.84	42,328.8
ANSALDO, 1936	d.m.	48 52 37.9545		08 27 08.74	188 26 21.58	RYAN	3.939 2517	8,694.642	28,525.67
(Y 698,051.92 ft.)		117 55 13.4487		24 00 45.07	203 59 28.36	DENNY	3.707 9279	5,104.202	16,746.04
(X 2,700,884.84 ft.)				340 00 34.55	160 02 16.38	O'TOOLE	3.906 9885	8,072.137	26,483.34
SMITH, 1936	d.m.	48 49 38.7320		39 40 14.19	219 38 40.50	RYAN	3.599 8744	3,979.920	13,057.45
(Y 680,057.86 ft.)		117 54 11.5761		104 40 35.08	284 38 31.83	DENNY	3.538 0047	3,451.475	11,323.71
(X 2,705,709.64 ft.)									

Note: Y and X are rectangular coordinates on the North Washington System of Plane Coordinates from Lambert Projection Tables by USC&GS, 1935.

FIG. 3.—GEOGRAPHIC POSITION SHEET

example, referring to Table 1, for an observed 500-ft measurement at 32° F, with 200 ft of the tape lying on a flat surface (supported throughout) and 300 ft of catenary, under 15 kg of tension, the correction would be $\frac{2}{5}(-05) + (-15) = -0.17$, or $500.00 - 0.17 = 499.83$ ft as the true distance.

TABLE 1.—Corrections for 500-Foot Tape in Hundredths of Feet (Two Supports; Tension = 15 Kilograms)

Length	Corrections for the Following Temperatures, in Degrees Centigrade:										
	−10	−05	00	05	10	15	20	25	30	35	40
Degrees Fahrenheit	14	23	32	41	50	59	68	77	86	95	104
25	01	01	00	00	00	00	00	00	00	00	01
50	01	01	00	00	00	00	01	01	01	02	02
75	02	01	00	00	00	01	01	01	02	02	03
100	02	02	01	01	00	01	01	01	02	03	03
125	02	02	02	01	00	00	01	02	02	03	04
150	04	03	03	02	01	00	01	02	03	04	05
175	06	05	04	03	02	01	00	01	02	03	04
200	08	06	05	04	03	02	01	00	02	03	04
225	10	08	07	06	04	03	02	01	01	02	03
250	12	10	09	07	06	04	03	02	00	01	03
275	15	14	12	10	08	07	05	04	02	00	01
300	19	17	15	13	11	09	08	06	04	02	00
325	21	20	18	16	14	12	10	08	06	04	02
350	26	24	22	20	18	15	13	11	09	07	05
375	31	28	26	24	22	20	18	15	13	11	09
400	36	33	31	29	27	24	22	20	17	15	13
425	43	39	38	36	33	30	28	26	23	20	18
450	49	46	44	41	38	35	33	30	28	25	22
475	57	54	51	48	45	42	40	37	34	32	29
500	65	62	59	56	53	50	47	44	41	38	35
500*	−11	−08	−05	−02	+01	+04	+07	+10	+13	+16	+19

* Tape supported throughout.
Note.—Values above the heavy line are plus; those below are minus.

Transit angles were measured by the repetition method, the observer recording one set of $\frac{3D}{R}$, and closing the horizon.

Variations in the method of third-order co-ordination included the following: (*a*) Triangle closure at the control station, in cases where cuts could be obtained only from two second-order stations; and (*b*) observation of one line of a three-point-fix reciprocally and holding the direction of that line as fixed, where it was impossible to obtain a cut to a fourth object for a check. Such variations, although seldom used, made possible consideration of the "fix" as a "check-position."

Third-order triangulation and traverse control extended over 406 sq miles, and a total of 1 934 points were co-ordinated by surveys of this accuracy. Including the positions of the 384 monumented stations occupied, a total of 4 051 control points and cadastral corners were co-ordinated over the project.

Related Surveys

The presentation, in this paper, of material pertaining to related surveys performed simultaneously with control and co-ordination will necessarily be limited to the discussion of the economies in time and effort resulting from the easy access to local control.

Topography.—Topographic mapping of parts of the draw-down area through town sites and across tracts of relatively high valuations made full use of adjacent control, both horizontal and vertical. Plane-table sheets were prepared in advance with the grid co-ordinate system, flow-line traverse, and such natural objects as might be used for resection cuts, plotted on them. Control proved of inestimable value in mapping these isolated areas.

Photogrammetry.—A stereotopographic map extending throughout the length of the reservoir site had been prepared jointly by the U. S. Army Engineers and the U. S. Geological Survey in 1930. A limited degree of control for the stereoplotting had been provided by the latter organization by plane-table triangulation, and although a faithful representation had been made of the terrain as to the proper shapes and sinuosities of the contours, the displacement toward or away from the river of the higher contours, due to lack of control away from the river banks, left in doubt the actual amount of ownership invasion of the 1 310 contour. The terrain below 1 100 was often the extent of topographic delineation on the maps, but in general the series provided a valuable aid to the Bureau's triangulation reconnaissance.

Early in 1934 a "flight" of single-lens aerial photographs was made for the project by a private contractor. This flight extended 75 miles up stream from the dam, embracing the area of the so-called low-dam backwater. A unit of the triangulation party was detailed to signal points on the ground to appear in the aerial photos, and to perform sufficient picture-point identification work and ground control to assemble the contact prints adequately into a controlled mosaic. Control points were selected on the prints and co-ordinated by substantially the same methods as were used in co-ordinating cadastral corners, and the section corners were identified wherever possible and pin pricked by the retracement engineers. In a few instances radial plots were made from the photos to enlarge upon some feature of a special right-of-way problem. After identification had been marked on the photographs, they were handed to the right-of-way appraisal crew, and greatly aided this feature of the acquisition work. Later they were utilized in the development of the new highway and rural road system which eventually replaced the existing road system of the area to be flooded.

Hydrography.—Control surveys, both horizontal and vertical, found widespread use in the field surveys to determine backwater effect of the reservoir pool. Sounding ranges were co-ordinated quickly throughout the backwater reaches, and profiles and soundings placed on the project datum from adjacent circuits of precise levels.

Miscellaneous.—Special field surveys called for by the Right-of-way Division embraced those made necessary by inaccurate or obscure descriptions found in previous conveyances. Often large discrepancies involving excesses or

deficiencies, or perhaps overlapping tracts, would necessitate a retracement of State highway, rural road, and railroad rights of way, and would require that these tracts be related to the adjacent properties. More often, an obscure point of beginning, an unclosed description, or questionable orientation, would present the troublesome feature of a conveyance. Once carefully surveyed and related to the governing system of rectangular co-ordinates, a perpetuation of boundaries would result, and would permit, at any future period, the exact re-establishment of boundary markers subsequently lost or destroyed.

One interesting problem arose in the requirement for replacing long-span telephone crossings. The length of span and elevations of supporting towers were determined quickly by a three-point-fix theodolite observation, performing at the same time a double-zenith-distance set of observations. Elevations obtained by this method were reliable to within one foot of the true elevation.

Of no small degree of importance on the project was the exacting requirement, at the Grand Coulee dam site, for precise construction control. To provide adequate control for the building of the world's largest structure, the standard of accuracy was set at that of first-order triangulation and precise leveling. Location of the axis of the dam, tentatively fixed by the U. S. Army Engineers some years before and marked by them with two iron pipes, was adopted subsequently by the Bureau of Reclamation as a result of its exploration studies. Projected outside the works area with the precise theodolite, the axis was marked permanently and referenced by grouting standard bronze disk markers in granite outcroppings, at points commanding a full view of the construction area. The stations thus marked were designated "east axis" and "west axis," and formed the south side of the first quadrilateral figure (see Fig. 4). Station Alpha, already set on the east bank near the center of the contractor's future town site, and Station Osborne, on the west bank in the Government engineers' future city, completed the figure. The base line was measured from Station Alpha to Station Ferry, this latter point lying on the axis of the dam at an elevation somewhat below Station East Axis.

Four steps of the survey were accomplished in occupying the stations of this first quadrilateral figure: namely—(*a*) The 1 100-m base line was expanded into the scheme, (*b*) third-order directions were measured to the near-by cadastral corner signals, (*c*) directions of first-order accuracy were measured to 40 supplemental stations established for works area control, and (*d*) observations were made on adjacent first-order stations for fixing the geographic positions and azimuths with respect to the 1927 North American Datum.

A rigid least-squares adjustment of the works area control was next performed, and a grid system, using the axis of the dam as meridian, was devised for use over the area. Supplemental grid lines, parallel and normal to the axis, were established on the near-by hills at intervals varying from 400 to 600 ft, the taping being with invar tapes equipped with spring balances and thermometers. Resulting lengths were adjusted to values furnished for each tape by the U. S. Bureau of Standards. By "wiggling-in" in two directions, any of the intersecting lines might readily be reproduced as the dam construction progressed, and thus afford an easy means of holding form-setters' chaining errors to a minimum.

In the first-order leveling, two reciprocal river crossings were observed, and 16 permanent bench-marks were established throughout the works area for vertical control.

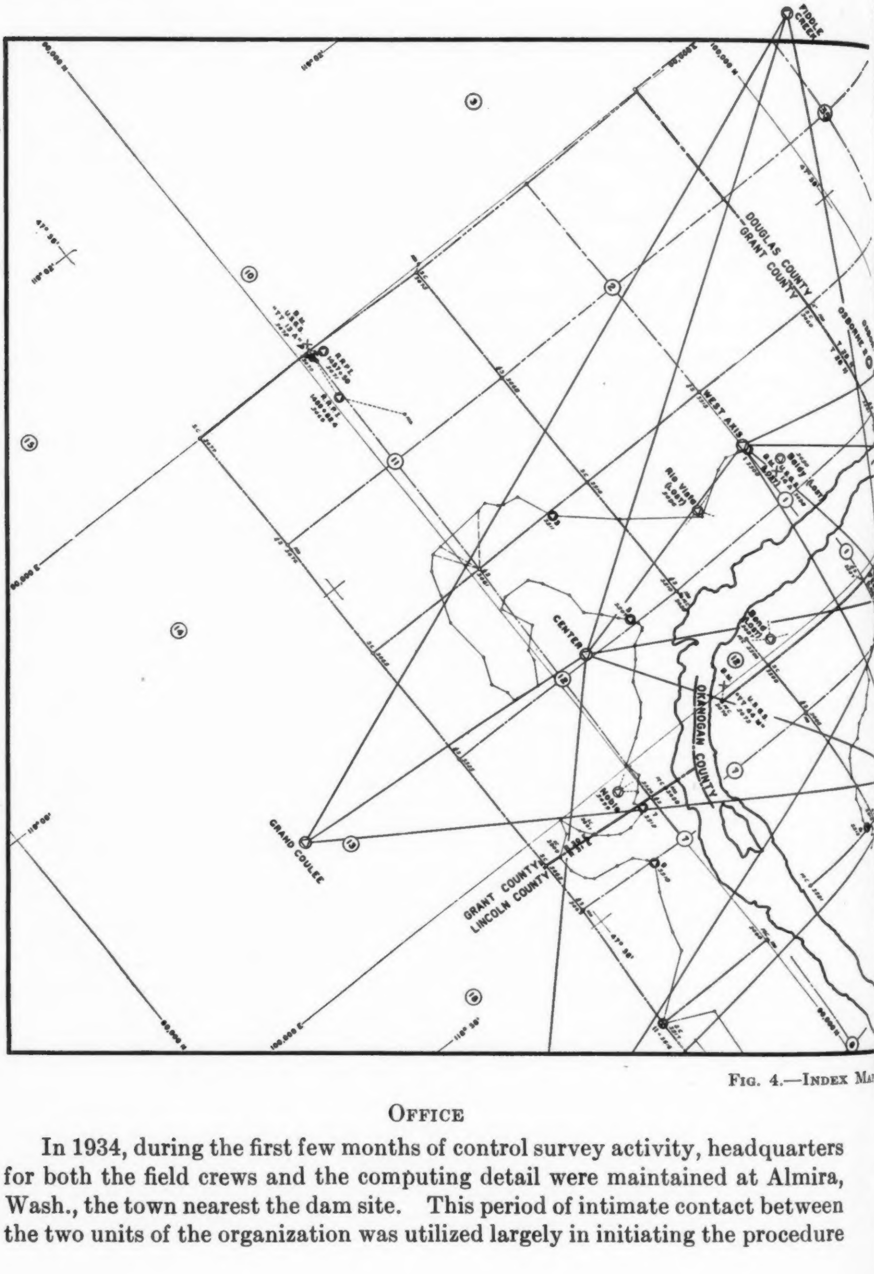
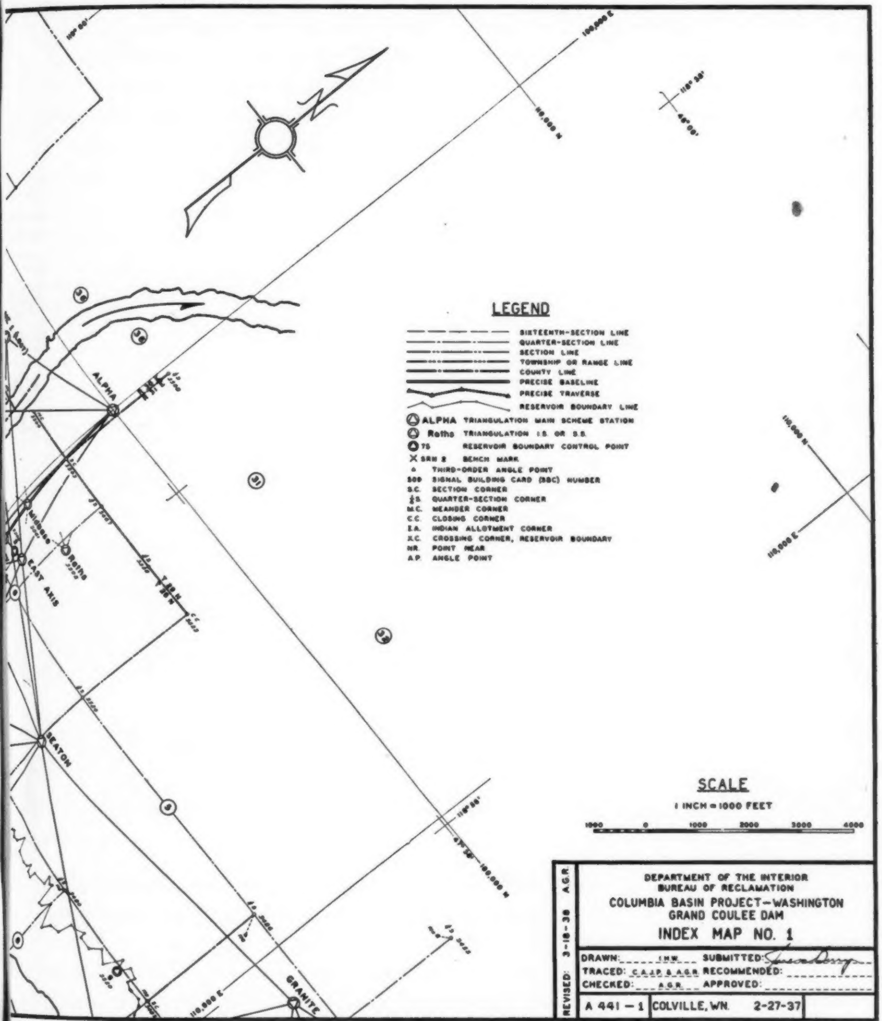


Fig. 4.—Index Ma

Office

In 1934, during the first few months of control survey activity, headquarters for both the field crews and the computing detail were maintained at Almira, Wash., the town nearest the dam site. This period of intimate contact between the two units of the organization was utilized largely in initiating the procedure

to be followed in the steps of converting field data to their final form of conveyancing plat. Later the two units were separated, the field office being moved successively to Davenport, Colville, and Coulee City, Wash.; and the computing detail was stationed permanently at the dam site (Coulee Dam,



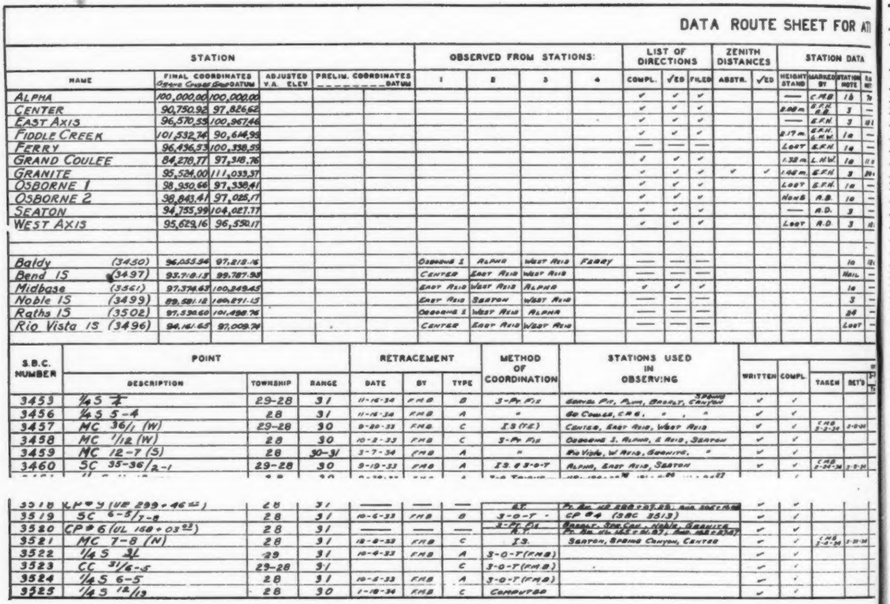
ICINITY OF GRAND COULEE DAM

Wash.). To facilitate the transmission of field data an expeditious system of data route sheets and index maps was devised (see Figs. 4 and 5).

Index maps were oriented to accommodate a maximum of area, with a minimum of overlap, without regard to cardinal orientation; 37 such maps were required over the reservoir area, and were numbered successively up stream from

the dam as A441-1, -2, etc. to -28 over the Columbia River area, A441-SP1 to -SP2 over the San Poil River, A441-SK1 to -SK5 over the Spokane River, and A441-KT1 to -KT2 over the Kettle River area.

Each cadastral corner or control point co-ordinated bore a serial number, first assigned to it by the signalman, and referred to as the Signal Building Card, or "SBC" number, of the point. For indexing, the "SBC" number was assigned to the lowest numbered index map and data route sheet of corresponding number, on which it first appeared. Thus, all data pertaining to each individually co-ordinated point, including its retracement record, signaling record, co-ordination list of directions, and its intersection or three-point-fix computation, together with any subsequent correspondence pertaining to



DATA ROUTE SHEET FOR AT[LAS MAP]

STATION: Name	Final Coordinates (Grand Coulee Grid Datum)		Adjusted V.A. Elev.	Prelim. Coordinates Datum		Observed from Stations: 1	2	3	4	List of Directions: Compl.	✓ed	Filed	Zenith Distances: Abstr.	✓ed	Station Data: Height Stand	Marked by	Station Note
Alpha	100,000.00	100,000.00								✓	✓	✓			—	C.M.B.	1b
Center	90,750.92	97,826.62								✓	✓	✓			2.08 m	E.F.H. A.B.	3
East Axis	96,570.55	100,967.46								✓	✓	✓			—	E.F.H.	3
Fiddle Creek	101,532.74	90,614.99								✓	✓	✓			2.17 m	E.F.H. L.H.W.	1a
Ferry	96,436.53	100,338.59								—	—	—			Lost	E.F.H.	1a
Grand Coulee	84,278.77	97,318.76								✓	✓	✓			1.32 m	L.H.W.	1a
Granite	95,524.00	111,033.37								✓	✓	✓	✓	✓	1.48 m	E.F.H.	3
Osborne 1	98,950.66	97,338.41								✓	✓	✓			Lost	E.F.H.	1a
Osborne 2	98,843.41	97,025.17								✓	✓	✓			None	A.B.	1a
Seaton	94,755.99	104,027.77								✓	✓	✓			—	A.D.	3
West Axis	95,629.16	96,550.17								✓	✓	✓			Lost	A.D.	3
Baldy (3450)	96,055.54	97,212.16				Osborne 1	Alpha	West Axis	Ferry	—	—	—					1a
Bend IS (3497)	93,718.13	99,787.93				Center	East Axis	West Axis		—	—	—					Hail
Midbase (3561)	97,374.63	100,249.45				East Axis	West Axis	Alpha		✓	✓	✓					1a
Noble IS (3499)	89,581.12	100,271.15				East Axis	Seaton	West Axis		—	—	—					3
Raths IS (3502)	97,530.60	101,498.76				Osborne 1	West Axis	Alpha		—	—	—					2a
Rio Vista IS (3496)	94,161.65	97,009.74				Center	East Axis	West Axis		—	—	—					Lost

S.B.C. Number	Point: Description	Township	Range	Retracement: Date	By	Type	Method of Coordination	Stations Used in Observing	Written	Compl.	Taken	Ret'd
3453	1/4 S 4	29-28	31	11-16-34	F.M.B.	B	3-Pt. Fix	Gravel Pit, Plum, Bradley, Spring Canyon	✓	✓		
3456	1/4 S 5-4	28	31	11-16-34	F.M.B.	A	"	Go Comes, C.R.G., " , "	✓	✓		
3457	MC 36/1 (W)	29-28	30	9-20-33	F.M.B.	C	I.S. (T.E.)	Center, East Axis, West Axis	✓	✓	C.M.B. 2-2-34	2-8-34
3458	MC 1/12 (W)	28	30	10-2-33	F.M.B.	C	3-Pt. Fix	Osborne 1, Alpha, E Axis, Seaton	✓	✓		
3459	MC 12-7 (S)	28	30-31	3-7-34	F.M.B.	A	"	Rio Vista, W Axis, Granite, "	✓	✓		
3460	SC 35-36/2-1	29-28	30	9-19-33	F.M.B.	A	I.S. & 3-0-T	Alpha, East Axis, Seaton	✓	✓	C.M.B. 2-24-34	3-5-34
3518	CP #5 (UP 299+46)	28	31	—	—	—	A.T.	[illegible]	✓	✓		
3519	SC 6-5/7-8	28	31	10-6-33	F.M.B.	B	3-0-T	CP #4 (SBC 3513)	✓	✓		
3520	CP #6 (UL 158+03)	28	31	—	—	—	3-Pt. Fix / A.T.	Bradley, Spr. Can., Noble, Granite / [illegible]	✓	✓		
3521	MC 7-8 (N)	28	31	12-8-33	F.M.B.	C	I.S.	Seaton, Spring Canyon, Center	✓	✓	C.M.B. 3-8-34	3-12-34
3522	1/4 S 31	29	31	10-4-33	F.M.B.	A	3-0-T (F.M.B.)		✓	✓		
3523	CC 31/6-5	29-28	31			C	3-0-T (F.M.B.)		✓	✓		
3524	1/4 S 6-5	28	31	10-5-33	F.M.B.	A	3-0-T (F.M.B.)		✓	✓		
3525	1/4 S 12/13	28	30	1-18-34	F.M.B.	C	Computed		✓	✓		

FIG. 5.—DAT[A]

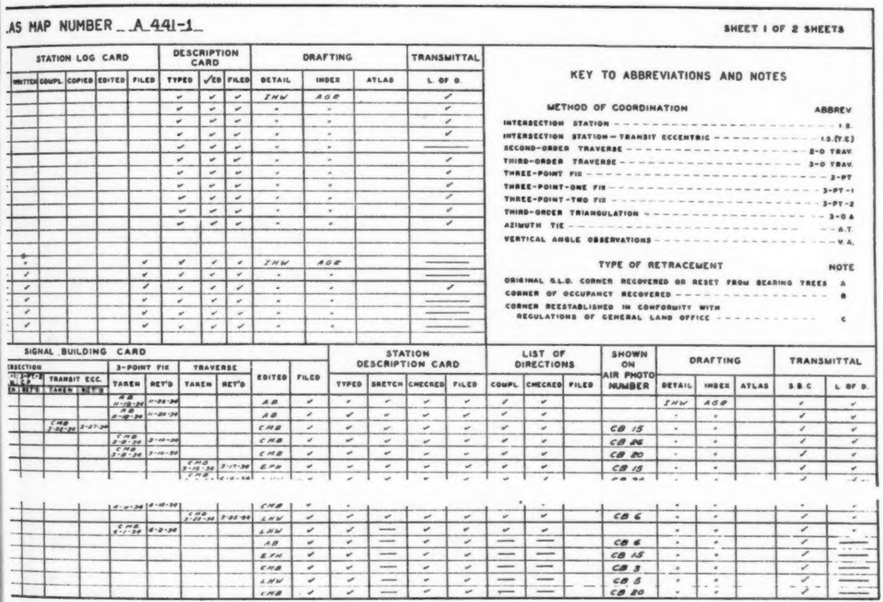
the point, were filed under the SBC number of each in the proper index map folder. Duplicate files were maintained in the field offices and at computing headquarters.

Advance sheets of the index map series were prepared in the field office as quickly as reconnaissance data were forthcoming. These sheets, locating approximately the positions of second-order stations, were compiled and blueprinted chiefly to aid the three-point party in their resection problems in the field.

The data route sheets were prepared at the field office in pencil as the work progressed. Upon completion of the work over that area, they were inked and sent with the index map to expedite the work of the computing force at the headquarters office. The data route sheet followed through each step of

the field and office procedure indicating, in addition to permanent station data, the identity of the field chief performing each step of the retracement, signaling, and co-ordination, as well as that of the abstract checker. In addition, the print number of the aerial photo upon which the picture-point identification had been made was indicated, and finally the drafting and transmission record was shown thereon.

The field office provided all final values for the geographic positions of first-order stations, and the vertical angle elevations of stations in the second-order nets of triangulation. Least-squares adjustments for this work were made generally during inclement weather by members of the field parties skilled in higher mathematics. Station description cards were typed in the field



Route Sheet

office for all monumented stations and co-ordinated points, and photostatic copies of these cards were prepared for transmission to other Governmental agencies. At headquarters office, field data were resolved to final co-ordinate values, and the individual parcel right-of-way plats were prepared for the Legal Division. At this office all second-order nets were adjusted by the direction method of least squares.

Two forms were devised by Bureau computers which deserve attention, one for the intersection station solution (see Fig. 6), and the other for solution of the resection, or three-point-fix problem (see Fig. 7). Each makes provision for carrying the computation through traverse courses beyond the "fix" point to the corner sought. Instructions for the use of these forms are to be found in Appendices I and II. The mathematical formulas used in the forms are not

new but the arrangement of terms is original and is believed to offer the utmost in speed where a calculating machine is available.

Intersection Computation Form.—At the headquarters office the data for coordinating most of the thousands of land monuments were received from the field offices in the form of "Lists of Directions," which were abstracted for

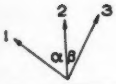
COORDINATION OF INTERSECTION STATION

Abstracted:			S.B.C. No.
REM.	F.S.B.		1538

							Y			X				
	224°	35′	21″6	1.										
	+ 29	51	05.4											
	254	26	27.0	INCHELIUM			479	159	04	2	639	946	87	
	N 74	26	27.0 E											
	44	35	23.1	2.										
	+305	25	36.9											
	350	01	00.0	ELBURN			493	607	21	2	654	189	41	
	S 9	59	00.0 E											
	80	41	25.9	3.										
	+ 48	45	28.5											
	129	26	54.4	DORR			482	086	61	2	657	805	36	
	N 50	33	05.6 W											
2	Y − 0	278	4370 x = −	255	899	85								
3	Y + 5	680	9446 x = +15	571	910	21								
A Coordinates 1·2	5	959	3816 x = 15	827	810	06	483	614	45	2	655	948	40	A
	Ecc. Bear. S 51° 00′ ″ E		Ecc. Dist.	0.75 ft.				− 0	47			+ 0	58	
	Twp. 32		Rge. 37	1/4 S. 3/10			483	613	98	2	655	948	98	*
	Check Cot.	0.278	735 4365 370	Check Bear.	N 74° 26′ 27.1 E		+ 4	455	41		+ 16	001	53	
	Check Cot.	5.680	2 5582 9646 1 5936	Check Bear.	S 9 58 59.9 E		− 9	992	76		+ 1	758	99	
2	Y − 0	278	4370 x = −	255	899	85			03				10	10
4	Y + 0	822	8263 x = +2	668	998	76								
B Coordinates 1·3	1	101	2633 x = 2	924	898	61	483	614	48	2	655	948	50	B
	Check Cot.	0.278	4366	Check Bear.	DO		+ 4	455	44		+ 16	001	63	
	Check Cot.	0.822	718 8246 472	Check Bear.	N 50 33 05.8 W		+ 1	527	87		− 1	856	86	
	Y		x											
	Y		x											
			x											

FIG. 6.—INTERSECTION COMPUTATION FORM

each triangulation station of second-order accuracy and contained, in clockwise order, the names of from four to seven second-order triangulation stations and of as many as a hundred or more cadastral points (third-order stations). Opposite each name appeared the angle which the direction to that signal had been observed to make with the direction to the initial or first point on the list. The initial point was always one of the second-order stations.



COORDINATION OF THREE-POINT FIX

Abstracted			S.B.C. No.
F.S.B.	R.E.M.		869

		Y	X
α = 75° 43′ 02.5″	1. SIX MILE	+ 371 020 42	+2 598 804 10
	2. NINE MILE	− 377 505 72	−2 596 855 69
	cot. α = +0.254 5741	A = −6 485 30	B = +1 948 41
		B cot α = + 496 01	−A cot α = +1 650 99
		O = −5 989 29	M = +3 599 40
β = 52° 23′ 25.4″	3. JENSEN	+ 387 668 73	+2 594 980 97
	2. NINE MILE	− 377 505 72	−2 596 855 69
	cot. β = +0.770 3710	C = +10 163 01	D = −1 874 72
		−D cot β = +1 444 23	C cot. β = +7 829 29
		L = +11 607 24	N = +5 954 57
$\frac{M-N}{L-O}$ = K = −0.133 8429 (726 / 297) = cot. bear. to 2		L−O = +17 596 53	M−N = −2 355 17
K^2+1 = +1.017 9139	$\frac{KO+M}{K^2+1} - \frac{KL+N}{K^2+1} = Q$	KQ = − 578 68	Q = +4 323 57
A — Bear. N 82° 22′ 36.0″ W	NINE MILE 2	+ 377 505 72	+2 596 855 69 — A
∠2·4 75 35 34.2	"Pt. Near"	− 376 927 04	−2 601 179 26
Bear. N 6 47 01.8 W	MITCHELL 4	+ 392 155 36	+2 599 367 86
Bear. N 6° 47′ 00.3″ W cot. = 8.406 9339 (7 0515 / 1176)		+ 15 228 32	− 1 811 40
Bear. diff. 1.5″ @ 15,300 ft. = 0.11 ft. ÷ sin ∠4·2 = 75° = 0.11 ft.			
75° 43′ 02.5″ / 104 16 57.5 1·2 checks α			
S 21° 54′ 21.5″ W cot. = 2.486 8304 (8813 / 509)	1-Pt. Nr.	− 5 906 62	− 2 375 16
N 82 22 36.0 W cot. = 0.133 8431 (8726 / 295)	2-Pt. Nr.	+ 578 68	− 4 323 57
N 29 59 10.6 W cot. = 1.733 0086 (0208 / 122)	3-Pt. Nr.	+10 741 69	− 6 198 29
52 23 25.4 2·3 checks β			
+ 97 37 24 / 183 24 12 / 281 01 36	"Point Near"	376 927 04	2 601 179 26
Bear. S 78° 58′ 20″ E	Dist. 152.50 ft.	−29 17	+ 149 68
1/4 S. 14/23	T. 29 R. 35	376 897 87	2 601 328 94 *
Bear.	Dist.		
	T. R.		
Bear.	Dist.		
	T. R.		
Bear.	Dist		
	T. R.		

FIG. 7.—THREE-POINT-FIX COMPUTATION FORM

After the office had completed least-squares adjustment work on the second-order net, each list of directions received two items which served to fix its position and orientation—namely, the Y and X co-ordinates of the occupied station and the grid azimuth of the initial direction. The adjustments of the second-order observations resulted in slight changes in the observed directions to all the second-order stations on a list, including the direction to the initial station. Thus, the initial station would have an adjusted direction slightly above or below its observed direction of $0° 00' 00''.00$; but there remained, as the point to which all unadjusted third-order observations on the list were referred, a direction of $0° 00' 00''.00$ which was no longer the direction to any real object. However, there was no confusion in this, as the fixed grid azimuth of the residual $0° 00' 00''.00$ direction was very easily computed and recorded at the head of the list. The grid azimuth of the "cut" from the occupied station to any signal is thus the sum of the grid azimuth of the initial direction and the direction to the third-order signal.

On the North Washington System of Plane Coordinates, a Lambert conformal conic projection, every cut may be represented by an equation of the form:

$$y - mx = b \qquad (1)$$

The plane co-ordinates of a third-order signal may be determined readily by a simultaneous solution of two equations which represent cuts from two second-order stations. Adopting this widely used principle of analytic geometry, the computation form devised for use on this project reduces the work of co-ordinating intersection stations to a routine requiring approximately 15 min of a computer's time for each point.

Each third-order signal was observed from three second-order stations. Only rarely were the three cuts exactly concurrent; usually they formed a small triangle of error. No attempt was made to determine the most probable position within this triangle. The co-ordinates of the point of intersection of the two cuts which intersected most nearly at right angles were adopted arbitrarily as final. Third-order observations were rarely in error by more than $3''$, and the cut from second-order station to third-order signal seldom exceeded 3 miles in length. Under such circumstances it appeared that the foregoing intersection point was usually within 0.15 ft of the point that would have been obtained by a least-squares adjustment of the three observations. In the computing procedure the co-ordinates of the points of intersection of the strongest and second strongest pairs of the three cuts are determined. The displacement between these two pairs of co-ordinates is also the length of the shortest side of the triangle of error. It is a valued indicator of the accuracy of the adopted co-ordinates and as such is conservative because it is probably somewhat greater than the actual error. The co-ordinates of the third and weaker vertex of the triangle of error are not computed. The form (Fig. 6) is equally adapted to use in resection problems and variations where only one or two cuts from second-order stations are available but additional observations have been made from the third-order signal to the second-order stations.

Three-Point-Fix Computation Form.—The form developed on this project for the solution of the resection, or three-point problem,[4] suggests several mechanical short cuts to the computer familiar with calculating machines. The example given in Fig. 7 required 20 min for the first computer and 15 min for the checker. Among the advantages of this form are: (1) One procedure takes care of all variations of the three-point problem; (2) necessity is eliminated for preliminary inverse computations between the fixed stations; (3) all computations are performed on one sheet; there is no need for scratch-paper computations; (4) warning is furnished if a fix close to the "great circle" is being computed; the quantities $L-O$ and $M-N$ both become very small and contain less significant figures than are required for K; (5) in its elimination of inverse computations, triangle computations, and all trigonometric functions except the cotangent, this form (Fig. 7) has many features in common with the Bureau of Reclamation form, Fig. 6 (as developed on the Columbia Basin Project) for computing intersection and resection stations, etc., in which an unknown point is quickly co-ordinated by simultaneously solving two or more equations of the form of Equation (1), every direction or "cut" on the plane co-ordinate system being exactly represented by such an equation. These similarities between the latter form and the three-point form contribute mutually to the development of speed in the work of an inexperienced computer who must become accustomed to all types of co-ordination problems. A discussion of the "fix strength index," as applied to this form (Fig. 7), is shown in Appendix II.

In one of the completed second-order nets, the third-order work disclosed a maximum side closure of 0.55 ft and an average of 0.21 ft. If one assumes the average length of sight to be 8 000 ft, the average closure of a three-point-fix or of a third-order intersection station is one part in 40 000, and the poorest closure one part in 15 000. There is no question that these accuracies were consistently attained. Adjustment statistics of the first-order and second-order nets are shown in Fig. 8.

Right-of-way plats (see Fig. 9) for the Legal Division of the Bureau were prepared on scales best suiting them for binding with conveyancing forms. Characteristic data appearing on the plat for each parcel were: (*a*) Representation of the value of θ, or mapping angle, from true north, (*b*) the recitation "Grid Bearings, Distances, and Coordinates are based on the North Washington System of Plane Co-ordinates," and (*c*) the Y and X co-ordinates of two adjacent "dry" monuments and the grid bearing and distance between these monuments.

Costs

A simplified cost keeping method was inaugurated early in connection with the control and co-ordination surveys, inasmuch as the new procedure was often subject to critical review, and further to facilitate a detailed analysis of the economies of the variations in procedure as the work progressed. The Bureau of Reclamation system of cost accounting, last revised in 1917 when the Bureau survey activities rarely, if ever, extended themselves beyond canal location and

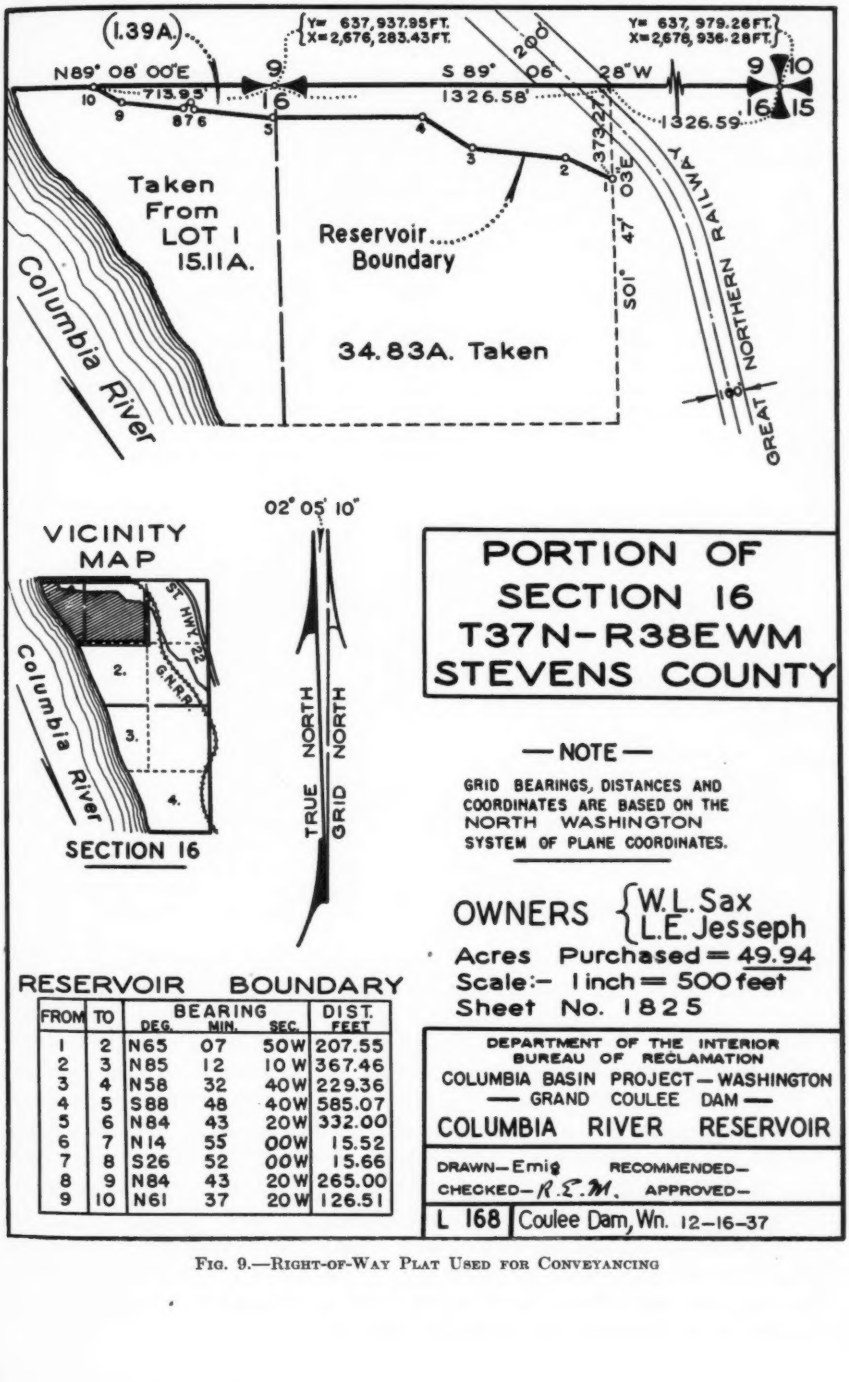
[4] For formulas, see *Special Publication No. 145*, U. S. Coast and Geodetic Survey, pp. 98 and 100.

UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

ADJUSTMENT STATISTICS OF TRIANGULATION NETS
COLUMBIA RIVER RESERVOIR --- COLUMBIA BASIN PROJECT

	FIRST-ORDER	SECOND-ORDER									
	Dam to Border	Breakdown figure C&GS to Dam	I	San Poil	II	Spokane	III	IV	V	Kettle	VI & VII
Total number of triangles	21	19	26	37	52	66	46	32	56	23	74
Number of triangles with plus closures	9	10	9	14	28	34	27	9	16	7	32
Number of triangles with minus closures	5	9	17	23	20	32	19	23	40	16	42
Number of concluded triangles	7	0	0	0	4	0	0	0	0	0	0
Average closure of triangles without regard to sign	1".32	2".435	1".572	2".11	2".42	1".64	2".38	2".05	2".10	3".38	1".97
Maximum closure of a triangle	3".08	7".02	7".58	6".65	6".61	4".45	7".40	5".27	6".09	6".95	7".64
Probable error of an observed direction	± 0".63	± 1".022	± 0".695	± 1".32	± 0".91	± 0".96	± 1".01	± 0".92	± 1".05	± 1".30	± 1".10
Mean error of an angle	± 0".70	± 1".113	± 1".250	± 1".14	± 1".09	± 1".147	± 1".727	± 1".448	± 1".503	± 2".202	± 1".443
Closure in length between bases after: (side and angle equations)	1/326,000	1/67,000	1/32,000		1/15,900				1/15,000		
or (side, angle, and azimuth equations)										1/97,800	
or (side, angle, azimuth, and latitude and longitude equations) have been satisfied.				1/59,400		1/65,000	1/26,000	1/22,000			1/800,000 (VI) 1/185,000 (VII)

FIG. 8.—TRIANGULATION STATISTICS



FROM	TO	BEARING DEG.	MIN.	SEC.	DIST. FEET
1	2	N65	07	50W	207.55
2	3	N85	12	10W	367.46
3	4	N58	32	40W	229.36
4	5	S88	48	40W	585.07
5	6	N84	43	20W	332.00
6	7	N14	55	00W	15.52
7	8	S26	52	00W	15.66
8	9	N84	43	20W	265.00
9	10	N61	37	20W	126.51

FIG. 9.—RIGHT-OF-WAY PLAT USED FOR CONVEYANCING

subdivisional surveys, was followed as the structure upon which to develop the new cost finding system:

Survey Cost Accounting: Grand Coulee Dam—Columbia Basin Project

Divisions

CR	Columbia River Reservoir
WA	Works Area (Precise Construction Control)
GC	Grand Coulee Balancing Reservoir
CB	Columbia Basin Economic Survey

Class Number

(667)	1 Cross-Section	(674)	10 Farm Units
(668)	2 Location	(675)	11 Triangulation—First-Order
(668)	3 Flow Line	(675)	12 Triangulation—Second-Order
(668*A*)	4 Retracement	(675)	13 Triangulation—Third-Order
(669)	5 Reconnaissance (other than triangulation)	(675)	14 Triangulation—Fourth-Order
		(675)	15 Precise Construction Control
(670)	6 Soil	(676)	16 Level Control, Precise
(671)	7 Subdivision	(676*A*)	17 Photogrammetric
(672)	8 Topographic	(676*B*)	18 Hydrographic
(673)	9 Trial Lines		

Detail

(*a*)	Reconnaissance (675 only)	(*o*)	Linear miles progress (axial)
(*b*)	Station marking	(*p*)	Cost per linear mile
(*c*)	Signal building	(*q*)	Square miles covered
(*d*)	Moving camp	(*r*)	Cost per square mile
(*e*)	Observing	(*s*)	Number of stations occupied
(*f*)	Base-line measurement	(*t*)	Cost per station occupied
(*g*)	Traverse	(*u*)	Number of targets set
(*h*)	Levels		Bench-marks set
(*i*)	Setting precise targets		Stations set
(*j*)	Picture point identification	(*v*)	Cost per station set
(*k*)	Miscellaneous	(*w*)	Number of points whose geographic positions or co-ordinates determined
(*l*)	Moving permanent headquarters		
(*m*)	Soundings, backwater studies		
(*n*)	Office computing, drafting, general (not to include abstracting of field records)	(*x*)	Cost per point determined
		(*y*)	Number of corners retraced
		(*z*)	Cost per corner retraced

Limited time for cost keeping by the field organization necessitated a system of simple structure, yet one which might be relied upon to give, when called for, any unit cost with accuracy. Segregations were made daily by each party chief, assigning in the proper column of the daily labor report the account number and detail symbol pertaining to his crew's work for that day. At the end of the month the office clerk added to the slip the amounts for salaries, ferry tolls, fixed office charges, material costs, etc. Transportation costs were computed on a cost-per-mile basis furnished the field office by the project cost accountant for each car or truck, and assigned at the field office to the proper job number on a mileage basis. Recapitulating accounts required only 3 or 4 hr of the clerk's time at the end of each month. Unit costs were computed and tabulated twice annually, June 30 and December 31. A partial tabulation of the field and field office costs of control and co-ordination surveys over the Columbia River reservoir site is given in Fig. 10.

Form 7-537 (May, 1930)

DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
COLUMBIA RIVER RESERVOIR

CR

Subject COSTS -- TRIANGULATION AND PRECISE SURVEYS

Sheet 1 of 6 sheets Date December 31, 1937 Computer A. G. R.

For other references see ________ Checked by ________

		THIS MONTH	THIS YEAR	TOTAL
2	Cross Section:			
k	Miscellaneous	--------	$ 17.71	$ 257.52
3	Flowline:			
g	Traverse	--------	$ 426.59	$ 2,009.92
h	Levels	--------	130.17	625.17
k	Miscellaneous	--------	3.38	95.30
o	Linear miles progress (levels and traverse)	--------	4.91	30.34
p	Cost per linear mile	--------	$ 113.39	$ 86.85
4	Retracement:	$ 10.40	$ 732.71	$ 3,244.19
k	Miscellaneous	--------	3.39	95.29
o	Linear miles progress	--------	13.6	273.8
p	Cost per linear mile	--------	$ 53.88	$ 11.85
y	Number of corners retraced	--------	29	798
z	Cost per corner retraced	--------	$ 25.27	$ 4.07
7	Subdivision:			
g	Traverse	$ 43.23	$ 43.23	$ 43.23
k	Miscellaneous	--------	1,991.94	1,991.94
n	Office	$ 12.39	12.39	12.39
8	Topographic:			
k	Miscellaneous	--------	$ 800.26	$ 940.19
11	Triangulation, First-Order:			
a	Reconnaissance	--------	--------	$ 865.56
b	Station marking	--------	--------	486.15
d	Moving camp	--------	--------	2,172.32
e	Observing	--------	$ 26.44	4,000.50
k	Miscellaneous	--------	7.38	91.29
l	Moving permanent headquarters	--------	86.42	339.69
n	Office	--------	766.40	4,700.18
o	Linear miles progress, axial	--------	--------	166.7
p	Cost per linear mile	--------	--------	$ 70.00
q	Square miles covered	--------	--------	2,714.7
r	Cost per square mile	--------	--------	$ 4.30
s	Number of stations occupied	--------	--------	35
t	Cost per station occupied	--------	--------	$ 333.38
w	Number of points whose G.P. determined	--------	--------	82
x	Cost per point determined	--------	--------	$ 142.30

Fig. 10.—Cost Sheet

Final costs per acre, based on the area over which retracement was required, varied from \$0.305 to \$0.345 for the different annual periods over which the work extended. Retracement costs, on a "cost-per-corner-retraced" basis, amounted to \$4.07 for the 3-yr period over which this activity was pursued. Many of these corners represented widely-scattered points in country exceedingly difficult to reach, found to be required for subdivision purposes many months after the first stadia retracement parties had gone over the area. Often too, the necessary search through county records would mount the retracement cost of the individual corners to as high as \$25. On the tributary streams where retracement work was totally completed by the triangulation party, 643 corners were recovered by the timber-cruiser method at a cost of \$2.48 per corner, or one-sixth the estimated cost of retracement by the stadia method. Likewise, the data shown for flow-line levels and traverse costs per mile are to be understood with the modification that they represent reruns of isolated sections found to be in error between triangulation control points, and are necessarily higher than costs attained over sections of continuous run.

Conclusion

Beyond the point of retracing the original General Land Office corners and other cadastral points and boundaries, the method developed presents a complete reversal of the classical, or usual "ground methods," of co-ordination. Although it establishes a precedent "beyond the previous knowledge of the Court," the method is believed well grounded in the legal elements affecting boundaries and adjacent properties, in that the mathematical determination of azimuths and distances by the inverse method is supported by the closely controlled allowable limits of closure, as well as the requirement for the position check of each point. These elements, coupled with the permanency of such a survey in connecting with a definite mathematical relationship to the Federal network, alone justify the procedure, and in light of the cost element attained, further justification for the procedure is apparent.

The writer is fully aware of the many limitations placed on work of this nature. Generally, topography and forest cover are primary factors in the economic consideration of control and co-ordination surveys; but linked inseparably with characteristics of the terrain are the following considerations: (1) The project must be situated close to points in the Federal network, or it must be of sufficient magnitude to warrant carrying in the control from some distant network; (2) there must be no compromise with the high quality of instruments and equipment required for the control part of such a survey; and (3) the selection of personnel to prosecute such a project must sedulously guard against the inclusion of any but those best qualified in their respective fields of this highly specialized branch of engineering.

Acknowledgments

Acknowledgment is made to Mr. R. E. McGowan, Assistant Bureau Engineer in charge of headquarters computing office, for his aid in developing the two computing forms, Figs. 6 and 7, with their respective instructions and commentaries, in Appendices I and II.

All the work of providing geodetic control and co-ordination over the Grand Coulee Dam Reservoir and Balancing Reservoir sites was undertaken with a field and office personnel varying from a maximum of 28 men to an average force of 15 men. The work was done by the U. S. Bureau of Reclamation on the Columbia Basin Project, with headquarters at Coulee Dam, all field work being under the direction of Frank A. Banks, Assoc. M. Am. Soc. C. E., as Construction Engineer.

All engineering and construction work of the Bureau is under the direction of R. F. Walter, M. Am. Soc. C. E., Chief Engineer, with headquarters at Denver, Colo., and all activities of the Bureau are under the general charge of John C. Page, M. Am. Soc. C. E., Commissioner, with headquarters at Washington, D. C.

APPENDIX I

Computation of Third-Order Intersection Stations

The computer about to co-ordinate a third-order intersection station will first examine a field map and determine the strongest and second strongest pairs of the three cuts, the criterion being the closeness with which the intersecting cuts approach right angles. The cut common to both pairs is designated "No. 1," the remaining cut of the strongest pair "No. 2," and the remaining cut of the second strongest pair "No. 3." Referring to the sample computation in Fig. 6, the procedure begins by entering the names of the three second-order stations, Inchelium, Elburn, and Dorr, on Lines 1, 2, and 3 in the order described. From the list of directions for Station Inchelium the following items are transcribed to the right and left of the space on the form where the station name appears:

(*a*) The Y and X co-ordinates of Inchelium.

(*b*) The grid azimuth, 224° 35′ 21″.6 (the grid azimuth of the 0° 00′ 00″.00 direction of the list).

(*c*) The direction to quarter-section corner between Sections 3 and 10; *viz.*, 29° 51′ 05″.4.

Addition produces the grid azimuth of the cut from Inchelium; written as a bearing it is N 74° 26′ 27″.0 E. Below, in the central part of the form on Line 1, Fig. 6, the equation of the Inchelium cut may be formed. The factor m in Equation (1) is 0.2784370, which is the cotangent of the angle 74° 26′ 27″.0. The sign of the mx-term is minus for cuts in the northeast and southwest quadrants and plus for cuts in the northwest and southeast quadrants. Substituting the Y and X co-ordinates of Station Inchelium in the equation,

$$Y - 0.2784370\, X = b \qquad (2)$$

the value of b is found to be − 255 899.85. It is advantageous to perform this substitution on a calculating machine in such a manner that the − mx-terms and Y-terms accumulate on the dials. Similar handling of the observations from Stations Elburn and Dorr completes the formulas on Lines 2, 3, 2, and 4

in the lower half of Fig. 6, as follows:

$$Y + 0.56809446\,X = b \ldots\ldots\ldots\ldots (3)$$

and

$$Y + 0.8228263\,X = b \ldots\ldots\ldots\ldots (4)$$

There is also space at the bottom of the form for a combination of Cuts 2 and 3, or of a possible fourth cut.

Proceeding with the use of Equations (2) and (3), it is evident that the Y-terms are readily eliminated by subtracting one equation from the other. The X-co-ordinate, obtained by machine division, is written on the right, in the X column. The corresponding value of Y results from a substitution of the X-value in either one of the original equations, preferably the one with the smaller cotangent. This substitution may be performed entirely on the computing machine. Following the same procedure with Equations (2) and (4), the co-ordinates of the $1 \cdot 3$ intersection appear in the lower part of the form.

The differences in Y and X between the $1 \cdot 2$ and $1 \cdot 3$ co-ordinates are 0.03 ft and 0.10 ft, respectively. The displacement between the two points of intersection is therefore 0.10 ft, usually written in the right-hand margin of the form. As this value lends assurance that the $1 \cdot 2$ co-ordinates are accurate within the specified limit of one part in 5 000, these are adopted as final.

Immediately below the $1 \cdot 2$ co-ordinate line, space is reserved for computing the co-ordinates of the monument proper if the signal is known to be eccentric. The eccentric distance from signal to monument, seldom exceeding 2 ft, along with carefully taken compass bearings observed from the signal to the monument and to the second-order stations, is taken directly from the Signal Building Card received from the field. On the sample computation form (Fig. 6), the bearing from the signal to the monument (S 51° 00′ E) has been referred to the Y-axis of the co-ordinate system by taking the angles between the compass bearings on the Signal Building Card and applying these to the reversed bearings in Sections 1, 2, and 3 in the upper part of the form.

Computation of the eccentric step should always be checked. In actual practice the work of the original computer appears in black ink, whereas that of the checker appears in red. For the purpose of differentiating in this paper the original computer's work is shown as tall, slope figures, whereas that of the checker's appears as small, vertical numerals. The checker does not examine the equations that his predecessor has used, but instead, computes the bearings to the $1 \cdot 2$ co-ordinates from Stations 1 and 2, and to the $1 \cdot 3$ co-ordinates from Stations 1 and 3. It is convenient to fold under the upper half of the form successively on the Lines *A–A* and *B–B*; the folded edge with the $1 \cdot 2$ or $1 \cdot 3$ co-ordinates uppermost may then be laid beneath the co-ordinates of Stations 1, 2, and 3 on these lists of directions and the sight differences in Y and X may easily be obtained by subtraction. Dividing the Y's by the X's gives the check cotangents which reproduce the foregoing bearings checked in Sections 1, 2, and 3, thus verifying all the computations leading to the adopted co-ordinates.

APPENDIX II

Computation of Third-Order Three-Point-Fix Stations

The computer about to co-ordinate a three-point-fix station will first examine a field map and, noting thereon the relative positions of the fix and the four second-order stations observed, make the selection of the three stations which will constitute the strongest fix. The fourth object, in this case Station Mitchell, is reserved for checking purposes. Hence, Stations Sixmile, Ninemile, and Jensen become Objects 1, 2, and 3, respectively, on the computing form (see Fig. 7). (If the point sought happened to lie within the fixed triangle Sixmile-Ninemile-Jensen, any one of the three fixed stations could be designated Object No. 1; No. 2 and No. 3 would follow in clockwise order.) The names of the four stations are written on the lines labeled 1, 2, 3, 2, 2, and 4 in the upper half of Fig. 7. The known Y and X co-ordinates of these stations are entered in the spaces to the right of their names. From the List of Directions the three angles $1 \cdot 2$ (α), $3 \cdot 2$ (β), and $2 \cdot 4$ are obtained by subtracting mentally and entering in the spaces provided on the form. The cotangents of α and β, taken from tables, are copied on the lines labeled "cot α" and "cot β." (Interpolation between the interval values given in the tables is an operation of only a few seconds on the calculating machine. For an angle greater than 90°, the cotangent is negative, of course.) Fig. 7 is self-explanatory as the computer proceeds down the right-hand side of the form to Line L–O and Line M–N. When these two terms are divided as indicated on the left-hand side of the form, they produce the quantity K. Directly below this quantity, $K^2 + 1$ may be developed. To the right, substituting values in either of the two expressions $\frac{(K\,O + M)}{(K^2 + 1)}$ or $\frac{(K\,L + N)}{(K^2 + 1)}$ produces Q, which is written in the space provided on the extreme right. Immediately to the left of Q is a space for the term $K\,Q$. Adding algebraically produces the position co-ordinates of the "Point Near," the name given to the point sought or that point from which the three-point observation was made. The minus signs before the Y and X co-ordinates of "Point Near" should be ignored at this time but will be found to be somewhat advantageous in the checking operation involving Station Mitchell. In cases where the value of K is greater than unity, Q may be a small quantity; in such instances Q should be evaluated to several places of decimals, as these significant figures will be essential to accuracy in developing the value $K\,Q$.

At this stage of the computation, the problem has been solved to the extent of determining the co-ordinates of the three-point-fix. An experienced computer performs this much of the computation in 12 min. The remainder of the form is used for checking both the field and office work and for resolving the co-ordinates of the "Point Near" through the short traverse step to the co-ordinates of the concealed monument, in this case the quarter-section corner between Sections 14 and 23.

Returning to the use of the fourth object, Station Mitchell, the computer makes the algebraic subtraction indicated where the co-ordinates of Station

Mitchell have been written. Dividing + 15 228.32 by − 1 811.40 results in a value which is the cotangent of the bearing to Station Mitchell; evaluated on the left this bearing is N 6° 47′ 00″.3 W. On the third line above this may be written the bearing to Station Ninemile, derived from K, which is the cotangent of the bearing to Ninemile; the quadrant in which this bearing lies is determined by reversing the algebraic signs of the quantities $K\,Q$ and K. Adding (clockwise) the observed angle 75° 35′ 34″.2 again produces a bearing to Station Mitchell, N 6° 47′ 01″.8 W, which is 1″.5 divergent from the Mitchell bearing previously obtained. This quantity (1″.5) is a measure of the accuracy of the field observations given in the List of Directions, provided: (*a*) The office solution has been free from error, (*b*) the rectangular co-ordinates of the four second-order stations are entirely accurate, and (*c*) the fourth object is so situated that it offers a strong check against the other three.

The computer who has completed his work to, and including, the line "Bearing Difference" (Fig. 7), finding the field data checking satisfactorily, proceeds to the bottom part of the form and makes the simple traverse computation to develop the co-ordinates of the quarter-section corner-stone. His initials, F. S. B., at the top of the form, indicate he has abstracted these co-ordinates.

A second computer checks the first man's work but follows a partly independent procedure which begins with the assumption that the co-ordinates of the "Point Near" are correct. He first verifies the three angles α, β, and 2·4. Next, in the center part of the form, he makes the subtractions 1-Pt.Nr., 2-Pt.Nr., and 3-Pt.Nr. in the Y and X columns. (This subtraction is performed most simply by folding under the top half of Fig. 7 above Line A–A on the sample and then successively placing the folded edge directly beneath the co-ordinates of Stations Sixmile, Ninemile, and Jensen where they are listed on the abstract sheets; the subtraction is thus made without copying down any figures.) Dividing the Y-values by the X-values produces cotangents and consequently, on the extreme left in Fig. 7, the bearings from "Point Near" to Objects 1, 2, and 3, respectively. Bearings 1 and 2 should reproduce α, and similarly, Bearings 2 and 3 should reproduce β, thus checking the office work that produced these co-ordinates for "Point Near." Below this line the second computer checks all the work of his predecessor.

Use of the Fourth Object to Check the Observations

On the line "Bearing Difference," the value 1.5 sec is of little aid in deciding whether a specified accuracy of one part in 5 000 is being met. Therefore, the estimated length of the line between "Point Near" and Station Mitchell, 15 300 ft, is multiplied by the sine of 1″.5 to give 0.11 ft, which value, in turn, is divided by the sine of the angle 4·2 to produce the ultimate index of accuracy, the figure on the extreme right-hand end of the line which, in this particular example, remains 0.11 ft. The experienced computer can usually determine the terms on this line mentally and without resort to tables. The last angle is not always 4·2 but, rather, is the angle between the direction to 4 and whichever of the other three objects will produce the angle nearest to a right angle. This entire procedure that involves the fourth object is entirely arbitrary;

the reason for performing it in this manner may be seen in the following review of three-point problem computation where a fourth object is available:

(*a*) Mr. W. F. Reynolds has shown[5] the method of combining all four directions in a least-squares adjustment. This ideal method, of course, is not intended for general engineering use.

(*b*) The suggestion of making two different solutions is made by the U. S. Coast and Geodetic Survey,[6] which omits one of the four objects in the first solution and omits a different object in the second solution.

(*c*) A variation of Method (b) is popularly used because it is quicker and apparently furnishes as faithful a check. This method may be illustrated by a solution of the Bureau of Reclamation sample three-point problem on Coast and Geodetic Survey Forms Nos. 655 and 25. The fourth object, Station Mitchell, is introduced only in the fourth triangle on Form No. 25. In that triangle, the second angle is obtained by subtracting the known fixed angle, Jensen-Ninemile-Mitchell, from the second angle of the second triangle. The common side of the fourth triangle disagrees by 0.13 ft with the side in the first and second triangles. Values greater than 0.13 ft would appear if the fourth triangle were Point Near-Jensen-Mitchell or Point Near-Sixmile-Mitchell. The fourth triangle is always chosen so that its first angle is as near a right angle as possible. It will be noticed that this is a practice identical to that on the Bureau of Reclamation form (Fig. 7) in which the value 0.11 ft was divided by the sine of the angle 4·2, which was the angle between Station Mitchell and whichever of the other three objects produced the nearest to a right angle. If a number of examples were solved by both methods it would be noticed that the final index value (0.11 ft) on the line "Bearing Difference" in the Bureau of Reclamation form is practically identical with the check value (0.13 ft) appearing in the fourth triangle of the method of triangles mentioned. No matter which of the two methods is followed, the index or check value obtained by using the fourth object as shown in these examples is believed to be as trustworthy an indicator of the accuracy of the field observations as can be obtained without a least-squares adjustment. The average of these index values for a large number of three-point problems in a given district appears to represent the probable error in displacement for the points co-ordinated. In any one example, however, its index value does not necessarily represent its probable error in displacement because, as previously noted, a fourth object will sometimes be so situated that it is incapable of checking all of the other three directions, and *vice versa*.

---

[5] *Special Publication No. 138* (Manual of Triangulation Computation and Adjustment), U. S. Coast and Geodetic Survey.

[6] *Special Publication No. 145* (Manual of Second and Third Order Triangulation and Traverse), U. S. Coast and Geodetic Survey.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

THE UNIT HYDROGRAPH PRINCIPLE APPLIED TO SMALL WATER-SHEDS

By E. F. Brater,[1] Jun. Am. Soc. C. E.

Synopsis

The unit hydrograph principle was first presented in 1932 by Leroy K. Sherman,[2] M. Am. Soc. C. E. Since then it has been developed into a usable method for surface run-off analysis. The most notable advance was the introduction of the distribution graph and pluviagraph in 1934 by Merrill M. Bernard,[3] M. Am. Soc. C. E.

Although the method has become quite generally accepted, the engineers and hydrologists who have made special studies in this field have suggested that further research be conducted to determine, more definitely, the limitations in the application of the unit hydrograph. One of the unsettled issues is the question of its usefulness on small streams.

It is hoped that the present investigation will provide a better understanding of the natural phenomena involved in the production of a unit hydrograph. The paper demonstrates that the unit hydrograph principle may be applied to small water-sheds, and that the distribution graphs and pluviagraphs are valuable tools for the analysis of surface run-off. The study further indicates that the unit hydrograph method is one of the best practical devices for predicting flood flows.

Introduction

The investigation described in this paper was conducted to determine whether the unit hydrograph principle is applicable to small streams and, if so, to determine its usefulness as a tool in the analysis of surface run-off.

Note.—Written comments are invited for immediate publication; to insure publication, the last discussion should be submitted by **January 15, 1940**.

[1] Instructor, Civ. Eng., Univ. of Michigan, Ann Arbor, Mich. (The study was undertaken while the writer was employed by the Appalachian Forest Experiment Station.)

[2] "Stream Flow from Rainfall by the Unit Hydrograph Method," by L. K. Sherman, *Engineering News-Record*, Vol. 108, 1932, pp. 501–505.

[3] "An Approach to Determinate Stream Flow," by Merrill M. Bernard, *Proceedings*, Am. Soc. C. E. Vol. 60, January, 1934, pp. 3–18.

The study is based on continuous records of run-off and rainfall taken from twenty-two small water-sheds by the Appalachian Forest Experiment Station.[4] The water-sheds lie in the high rainfall belt of the Southern Appalachians. They vary in area from 4.24 acres to 1 876.7 acres, and in cover type, from old-growth forest to complete denudation.

The first part of the investigation was devoted to the selection of unit hydrographs and the preparation of distribution graphs on the twenty-two streams. The method of selecting the unit hydrographs from the discharge records of the water-sheds is presented, with a general consideration of the principles involved in the selection. The various factors to be considered in the separation of surface run-off from ground-water flow are discussed, and a method of procedure for making this separation on hydrographs from small streams is suggested. Distribution graphs were derived from each of the unit hydrographs, and the graphs from each stream were superimposed upon one another to permit the selection of a composite distribution graph for the stream. A detailed study was made of the rainfall that produced the unit hydrographs for the purpose of determining more definitely the conditions that govern the formation of such graphs.

The second part of the work consists of some applications of the distribution graphs for the purpose of further testing the adequacy of the theory, and to indicate the value of the unit hydrograph as a tool in the analysis of run-off phenomena. The composite distribution graphs were analyzed for the purpose of disclosing any correlations between the shapes of the distribution graphs, especially the peak percentages and the widths of bases, with the physical characteristics of the water-sheds. The distribution graphs of three of the streams were then utilized in the construction of pluviagraphs for comparison with actual hydrographs of run-off. The pluviagraph studies led logically to a consideration of surface run-off coefficients. Finally, a method is submitted for the use of the unit hydrograph principle as a practical method of predicting surface run-off.

Basic Data

The investigation is based on rainfall and discharge records collected continuously for periods varying from two to three years. Fifteen of the twenty-two water-sheds are forested, two are void of all vegetation, and the remainder have varying types of vegetative cover. They are situated in three separate experimental areas, the streams of each group being designated by the name of the experimental area together with their number within the area.

Locations of the Installations.—The first group of seven installations is in or near the Bent Creek Experimental Forest which is about 11 miles south of Asheville, N. C. Four of these are forested, and one is a mixture of forest and adjoining agricultural land, whereas the other two are over-grazed pasture and abandoned farm land. The location of Bent Creek and the other two experimental areas is shown in Fig. 1. Descriptive data concerning all individual water-sheds are given in Table 1, the vegetal cover in each case being as follows:

---

[4] Appalachian Forest Experiment Station, U. S. Forest Service, Department of Agriculture, Asheville, N. C. (see "Engineering Aspects of the Influence of Forests on Mountain Streams," by Richard A. Hertzler, Jun. Am. Soc. C. E., *Civil Engineering*, August, 1939, p. 487).

Stream No.	Description
Bent Creek 1	Forest; oak-chestnut type; a high percentage of old-growth timber; the forested areas were heavily settled and farmed prior to 1900; no large-scale lumbering operations have been undertaken on this area except for cutting and clearing on the individual farms. Scattered areas were burned ftom time to time, prior to 1926.
Bent Creek 2	Forest; two-thirds of the standing trees, including all non-commercial trees, were cut in the winter of 1930–1931; heavy skidding operations at that time greatly depleted the litter layer.
Bent Creek 3	Over-grazed pasture and abandoned old fields; sheet erosion well advanced.
Bent Creek 4	Approximately 50% forested, and 50% cultivated land and abandoned old fields.
Bent Creek 5 and 6	Forest; second growth of the oak-chestnut type.
Bent Creek 8	Abandoned old fields; eroded.
Copper Basin 1 and 2	Forest of the oak-chestnut type; this region has a relatively thin layer of soil, underlain by shales and sandstone.
Copper Basin 3 and 4	Grass-land; sheet erosion and some gully ng in progress.
Copper Basin 5 and 6	No cover; top-soil completely removed and terrain deeply gullied.
Coweeta 1 to 9	The primary vegetation of all the Coweeta watersheds is second-growth forest of the oak-chestnut type, with scattered areas (less than 25% of the whole) of old-growth timber. The entire area was lumbered commercially as long ago as 1919.

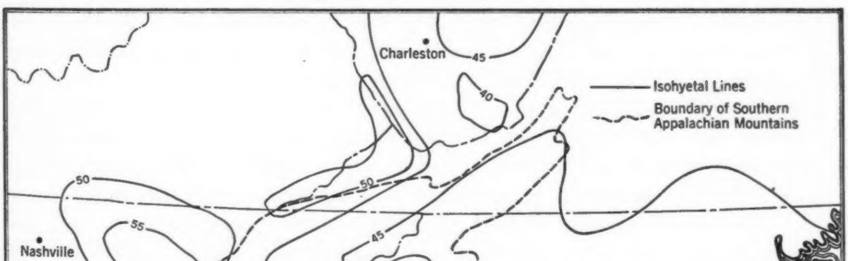

FIG. 1.—MAP OF APPALACHIAN REGION

TABLE 1.—DESCRIPTION OF WATER-SHEDS IN THE APPALACHIAN REGION (FIGURE 1)

Stream No.	Area, in acres	Discharge, in Cubic Feet per Second per Square Mile		Elevation above Sea Level (Atlantic Datum), in Feet	
		Maximum	Minimum	Weir	Maximum of water-shed
	(1)	(2)	(3)	(4)	(5)
(a) Bent Creek Water-Sheds					
1	48.6	41.5	0.075	2 080*	2 450
2	17.3	84.3	0.055	2 020*	2 680
3	10.78	335.0	0.125	2 050*	2 150
4	773.95	38.2	0.189	....†	....
5	90.4	61.1	0.06	2 350*	3 100
6	71.5	68.3	0.23	2 400*	3 100
8	16.25	188.3	....	2 160*	2 220
(b) Coweeta Water-Sheds					
1	38.77	41.60	0.35	2 300*	3 250
2	30.78	55.55	0.11	2 350*	3 314
4	10.18	83.03	0.29	2 750*	3 000
5	4.24	88.99	0.81	2 300*	2 500
(b) Coweeta Water-Sheds (*continued*)					
6	21.88	66.28	0.25	2 300*	2 650
7	145.45	37.12	0.71	2 400*	3 491
8	1 876.7	57.76	0.75	2 300‡	5 097
9	1 787.9	67.58	0.72	2 250‡	4 887
10	212.3	36.57	0.87	2 450§	3 600
(c) Copper Basin Water-Sheds					
1	19.0	144.4	0.005	1 632*	1 947
2	88.7	164.0	0.14	1 635*	2 492
3	6.0	832.0	0.00	1 528*	1 645
4	6.7	1 094.7	0.16	1 554*	1 698
5	15.6	1 263.2	0.04	1 695‖	1 821
6	4.7	1 433.9	0.00	1 719*	1 817

* Right-angle, V-notch. † 5-ft trapezoid. ‡ 12-ft trapezoid. § 120° V-notch. ‖ 3.5-ft trapezoid.

Coweeta Experimental Forest (see Table 1(*b*)) is about 15 miles south of Franklin, N. C., and 6 miles north of the Georgia State line. The two larger water-sheds of this group (Stream No. 8 and Stream No. 9) include the entire experimental area. All of the others lie within the boundaries of these two. The entire area is covered with second-growth forest.

The remaining six water-sheds are in the southeast corner of Tennessee, within or near the area known as the Copper Basin (see Table 1(*c*)). This region derives its name from the copper-mining and smelting activities in Copperhill and Ducktown, Tenn., about which the area centers. The Copper Basin consists of approximately 8 sq miles of completely denuded land surrounded by a zone of grass-land the area of which is 22 sq miles. The denudation was started by cutting the larger trees, but the principal damaging factor was the sulfur fumes resulting from the original open roast-heap method of smelting. The denuded area is deeply gullied and all top-soil is washed away. The grass-land has suffered from sheet erosion and is in early stages of gully formation. Two water-sheds were gaged in each of these zones. Two others are located just west of the Copper Basin in the Cherokee National Forest.

Methods of Measurement.—Stream flow was gaged by means of standard sharp-crested weirs, right-angle V-notch weirs being used on the smaller streams and trapezoidal weirs on the larger ones. Heads over the weirs were recorded continuously by float gages. Each weir was set accurately in a concrete dam,

which was extended to rock bottom in all cases. Stilling basins behind all the weirs were more than adequate to satisfy requirements for proper contraction of the jet and for reducing the velocity of approach to a negligible quantity. The type of weir used in each case is indicated by footnotes in Table 1.

Rainfall was measured by means of standard Weather Bureau rain gages, with a smaller number of recording gages. There were twenty-seven standard rain gages and five recording gages on the Bent Creek water-sheds, twenty-three standard and six recording gages on the Copper Basin water-sheds, and sixty-three standard and six recording gages on the Coweeta water-sheds. Considering the total area of all the water-sheds, the average area per rain gage was thirty-seven acres. The gages were arranged to give the best possible sample, taking into account aerial distribution, elevation, and aspect. The average precipitation was computed by the Thiessen method.[5]

The Preparation of Unit Hydrographs and Distribution Graphs

The principle underlying the unit hydrograph method may be stated briefly as follows:[6] For any water-shed, the surface run-off resulting from a unit storm is distributed, according to time, in a characteristic manner, and this distribution is independent of the intensity of the rainfall.

A unit storm is a rain of an intensity sufficient to produce surface run-off which occurs within the period of rise of the hydrograph. The period of rise is the time from the beginning of surface run-off to the occurrence of peak discharge.

Definitions.—Since the terminology that is applied to this phase of hydrology is not entirely uniform, the following terms are defined according to their usage in this paper.

Surface Run-Off.—That portion of the precipitation which reaches the stream by means of overland flow.

Period of Surface Run-Off.—The time intervening between the appearance of surface run-off at the gaging station and the time when all surface run-off has passed the station. The period of surface run-off is equal to the period of overland flow plus the time required for the remaining channel storage, derived from overland flow, to pass the gaging station.

Period of Rise.—The time intervening between the beginning of surface run-off and the occurrence of the peak discharge.

Infiltration Capacity.—The maximum rate at which a given soil, when in a given condition, can absorb falling rain.[7]

Surface Run-Off Coefficient.—The percentage of the total rainfall on a water-shed that appears at the gaging station as surface run-off.

Unit Storm.—Isolated rainfall falling at an intensity greater than the infiltration capacity and having a duration equal to or less than the period of rise.

Unit Hydrograph.—A hydrograph of surface run-off resulting from a unit storm.

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[5] *Journal*, New England Water Works Assoc., March, 1924, p. 26.

[6] Geological Survey Water-Supply Paper 772.

[7] "Surface Run-off Phenomena, Part I—Analysis of the Hydrograph," by Robert E. Horton, *Publication 101*, Horton Hydrological Lab., Voorheesville, N. Y.

Distribution Graph.—A graph having a time base equal to that of the unit hydrograph from which it was derived and having ordinates that represent the percentage of total surface run-off that occurs in certain selected equal time intervals. It provides a graphical representation of the characteristic distribution of surface run-off from a water-shed.

Selection of the Unit Hydrographs.—The first step in the preparation of the distribution graphs was the selection of unit hydrographs from the continuous run-off records. The method was to select isolated, clean-cut surface run-off

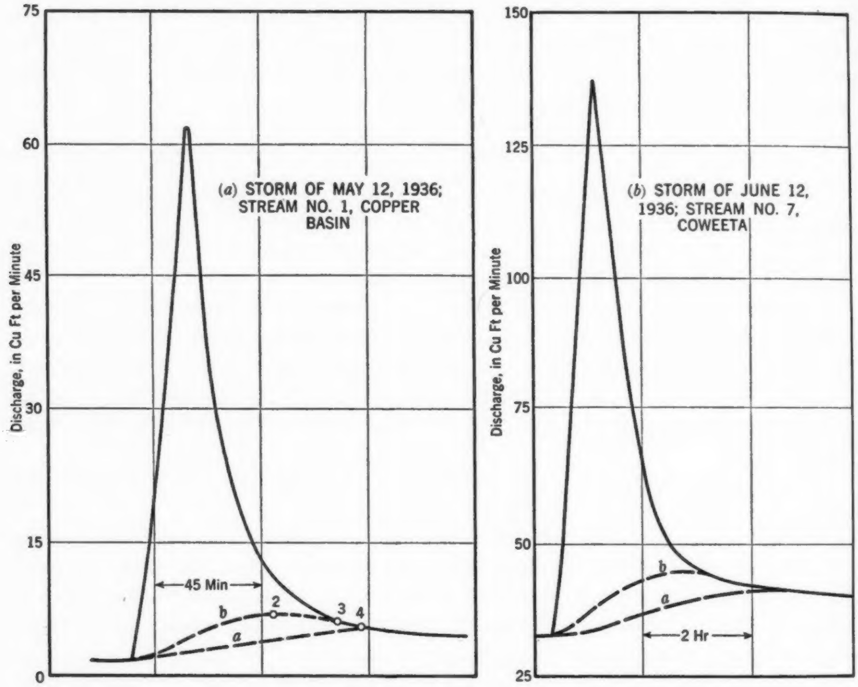


FIG. 2.—TYPICAL HYDROGRAPHS

graphs that occurred at a time of low flow (see Fig. 2). If insufficient graphs of this nature were available, those which were affected by small rains before or after the main one were used. In such a case the main graph was separated from its "parasite" after a careful study of the normal rising or receding sides of other surface run-off hydrographs from the same stream.

This procedure was necessitated by a lack of knowledge of the type of rains that would produce unit hydrographs. In the light of the knowledge gained from the work, it is possible to select the unit storms from the rainfall records by selecting intense isolated rains, the duration of which was less than the period of rise indicated by the area and nature of the water-shed. The only advantage of selecting the unit storms from the rainfall records rather than from stream-flow

records occurs when continuous stream-flow records in the form of hydrographs are not available.

The Separation of Surface Run-Off from Ground-Water.—The second step in the preparation of the unit hydrographs was the separation of surface run-off from ground-water contribution. Under these ideal conditions (isolated storms occurring at times of low flow) this procedure became relatively simple. The separation was made first as indicated by Base Line *a*, Fig. 2(*a*). A study of the ground-water build-up during rainfall later indicated that for most streams a base, such as Base Line *b*, was more nearly correct. In the case of Stream No. 1, Copper Basin, distribution graphs were prepared for both sets of base lines and it was found that the difference was small. An investigation of the hydrographs produced by rains of such intensities that no appreciable surface run-off occurred has also indicated that the general shape of the line separating ground-water from surface run-off is in the nature of Base Line *b*. The following two considerations lend weight in favor of the accuracy of this observation.

First, the rate of ground-water contribution depends upon the average slope of the water-table at the shores of the stream. Obviously, the ground-water elevations adjacent to the stream must rise more rapidly than the stream elevations to cause the rate of ground-water inflow to increase during the course of surface run-off. This is true in the case of the small water-sheds under consideration, where the rainfall nearly always covers the entire water-shed. It seems logical to assume that the maximum rate of ground-water contribution (Point 2, Fig. 2(*a*)) will occur at a time after the elevation of the water in the stream has finished its rapid decline, while the adjacent ground-water is still nearly at a maximum elevation. This point occurs near the time when overland flow ends, and during that time when the surface run-off at the gaging stations consists only of channel storage.

Second, it is inconceivable that the ground-water discharge could vary in an erratic or jerky manner. Similarly, the effect of channel storage cannot be thought of as ending abruptly. It follows, therefore, that the line of separation must be a smooth curve tangent to the actual hydrograph both where it leaves and where it rejoins it.

If continuous records of ground-water elevations were taken in conjunction with records of water elevation in the stream, it would be possible to determine the exact nature of this line of separation with considerable accuracy.

Although the preceding comment concerns especially the conditions on small water-sheds, the same general line of reasoning may be applied to larger water-sheds. However, in the latter case, unusual conditions may arise which require special consideration. For instance, it frequently happens that rain occurs only in the upper reaches of a water-shed and misses entirely the region where the gaging station is situated. When the stream rises at such a station, there must be a temporary reversal in the ground-water gradient and the ground-water contribution could conceivably be negative for a short interval. Even in this case, the volume of water required in most water-sheds to build up the underground water to the level of the stream would be quite small. Furthermore, a portion of the water flowing in the stream is ground-water contribution

from the region in which the rain occurred, and it would be a very extreme case if this volume were not more than sufficient to supply the deficiency in the lower reaches. It can safely be assumed that, in all cases, the line of separation will have a minimum slope of zero and generally will slope upward during the period of overland flow.

On the basis of tests by the Appalachian Forest Experiment Station, thus far, it may be stated with considerable assurance that the line of separation for small water-sheds is a reverse curve of the nature of Base Line *b*, Fig. 2(*a*). The line is quite definitely fixed by the two points of tangency (Points 1 and 3) and by the maximum ordinate (Point 2).

Generally, even in the case of storms producing overlapping unit hydrographs, the point where surface run-off begins (such as Point 1, Fig. 2) is quite definite.

In the case of the hydrograph shown, the point where surface run-off ends (namely, Point 3) was selected because at this point the graph flattens out rather suddenly. Such a point is not always evident and its selection requires considerable judgment. Although this point cannot be chosen definitely, the maximum time that surface run-off continued after the rainfall ended usually may be selected with assurance, as for instance the time from the peak to Point 4, Fig. 2. This information becomes extremely important when applied to complicated hydrographs resulting from a series of rains, in which instance the selection of this point becomes much more difficult. In cases of this nature, studied by the writer, the lengths of base lines determined by different persons have varied by as much as 100 per cent. By utilizing the knowledge gained from the unit hydrograph, this difference would be more of the order of 5 per cent.

The high point (such as Point 2, Fig. 2(*a*)) probably occurs near the end of overland flow, or about half-way between the peak of the hydrograph and the end of surface run-off. Until more quantitative results are available, the height of this point must be a matter of judgment based on a study of the flow characteristics of the particular stream.

The Construction of the Distribution Graphs.—Having separated groundwater from surface run-off, the unit hydrographs are represented by the ordinates between the line of separation and the actual hydrograph. The unit hydrographs thus determined were then divided arbitrarily into time intervals, and the average rate of surface run-off was determined for each interval. These values were totaled and the percentage of the total occurring during each of the periods was computed. The resulting percentages are a numerical representation of the distribution graphs. Time intervals varying from 2.5 min to 30 min were used, the object being to represent the shape of the graph faithfully with a minimum of labor.

It should be noted that the magnitude of the percentages making up a distribution graph depends on the time interval selected. If 7.5-min intervals instead of 15-min intervals were chosen in a given case, it is evident that the percentages would be half as great. Therefore, for the purpose of comparison, all the distribution graphs were converted to a common time interval. For

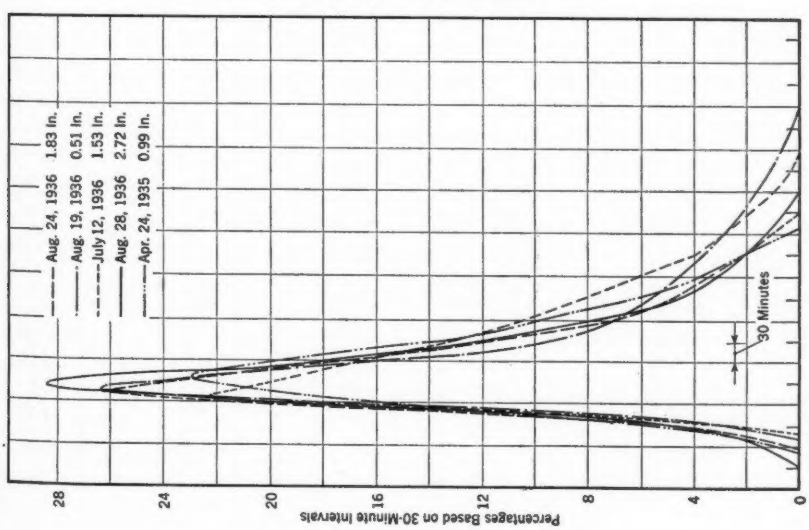


FIG. 3.—DISTRIBUTION GRAPH; STREAM NO. 9, COWEETA WATER-SHED; 1 788 ACRES; FORESTED

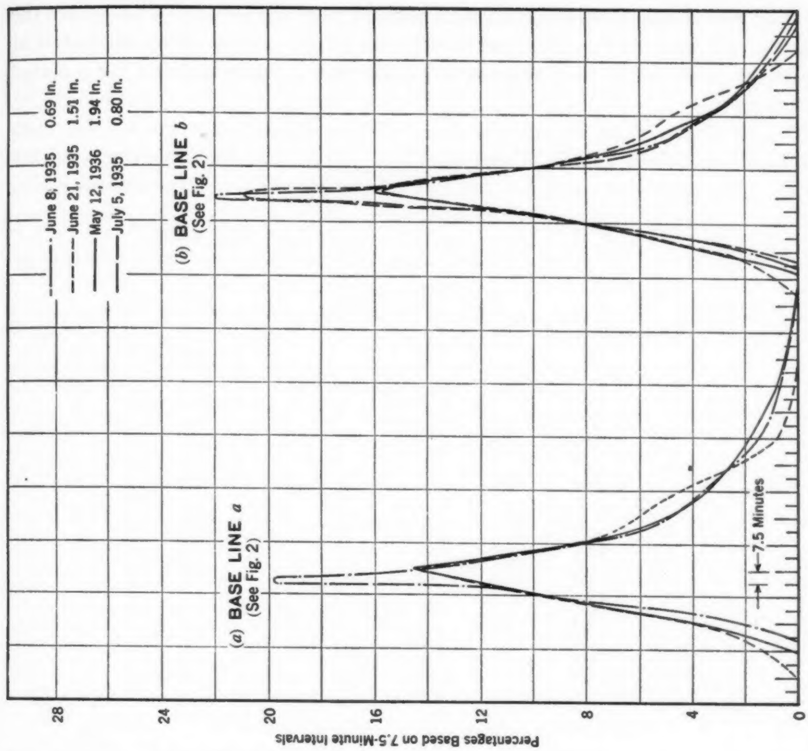


FIG. 4.—DISTRIBUTION GRAPH; STREAM NO. 1, COPPER BASIN WATER-SHED; 19.0 ACRES; FORESTED

example, percentages based on 15-min periods were divided by 6 to reduce the data to a 2.5-min basis. If these results are plotted, they must be spaced at 15-min intervals, since they represent the average 2.5-min interval for a period of 15 min.

The final step was to superimpose all the distribution graphs for each stream as nearly as possible on one another. Figs. 3 and 4 show the resulting clusters of characteristic graphs for two of the streams. A composite distri-

TABLE 2.—DATA FOR DISTRIBUTION GRAPHS

Interval No.	Stream Numbers:									
	1	2	3	4	5	6	7	8	9	10
	(a) BENT CREEK WATER-SHEDS									
*T** =	7.5	7.5	5.0	7.5	15.0	7.5	....	5.0	....	....
1	1.6	2.0	2.5	1.4	0.4	3.0	....	10.0	....	....
2	6.0	19.0	14.0	3.2	7.4	14.0	....	19.0	....	....
....	....	(52.0)†	....	....	....	(20.0)†	....	....	....	....
3	12.8	33.0	41.0	5.2	21.6	17.0	....	48.0	....	....
....	(20.4)†	....	(52.5)†	....	(34.0)†	....	....	(80.0)†	....	....
4	18.4	16.0	15.0	8.0	25.4	13.0	....	7.0	....	....
5	13.0	8.0	9.0	12.0	14.0	10.0	....	5.0	....	....
6	10.6	6.5	6.0	17.4	7.0	8.0	....	3.5	....	....
....	....	....	....	(18.7)†	....	....	....	....	....	....
7	9.6	4.5	4.5	15.2	4.6	6.0	....	2.5	....	....
8	6.8	3.5	3.0	9.0	3.6	5.0	....	2.0	....	....
9	4.4	2.5	2.0	6.2	2.8	4.0	....	1.5	....	....
10	3.6	2.0	1.5	5.4	2.8	3.6	....	1.0	....	....
11	2.8	1.5	1.0	4.5	2.4	3.1	....	0.5	....	....
12	2.4	1.0	0.5	3.7	2.0	2.7	....	....	....	....
13	2.0	0.5	....	2.8	1.6	2.4	....	....	....	....
14	1.6	....	....	2.0	1.6	2.0	....	....	....	....
15	1.3	....	....	1.4	1.2	1.6	....	....	....	....
16	1.1	....	....	1.0	0.8	1.3	....	....	....	....
17	0.8	....	....	0.7	0.4	1.0	....	....	....	....
18	0.6	....	....	0.5	0.4	0.8	....	....	....	....
19	0.4	....	....	0.3	....	0.6	....	....	....	....
20	0.2	....	....	0.1	....	0.5	....	....	....	....
21	....	....	....	....	....	0.3	....	....	....	....
22	....	....	....	....	....	0.1	....	....	....	....
	(b) COWEETA WATER-SHEDS									
*T** =	15.0	15.0	....	15.0	7.5	15.0	15.0	30.0	30.0§	15.0
1	2.6	1.2	....	0.9	1.1	8.0	0.6	0.5	0.6	1.8
2	10.9	4.8	....	9.3	6.9	27.6	3.4	5.7	2.4	6.4
....	....	....	....	....	....	(42.5)†	....	....	....	....
3	21.7	12.2	....	31.6	18.5	28.7	6.4	13.4	9.0	12.2
....	(29.1)†	....	....	(46.8)†	(34.2)†	....	....	....	....	....
4	23.9	24.0	....	33.4	30.0	14.4	20.8	20.9	25.4	17.8
....	....	....	....	....	....	....	(21.5)†	(22.1)†	(28.6)†	....
5	16.1	30.4	....	10.5	16.5	9.0	19.6	17.7	23.3	20.0
....	....	(35.2)†	....	....	....	....	....	....	....	(21.0)†
6	9.6	15.7	....	6.2	8.9	5.9	14.2	13.4	14.5	15.2
7	6.4	6.8	....	4.1	6.0	3.7	10.0	9.7	9.7	11.3
8	4.3	3.1	....	2.3	4.3	2.0	6.8	7.2	5.9	7.0
9	2.7	1.4	....	1.2	2.9	0.7	4.6	4.7	3.9	3.8
10	1.3	0.4	....	0.5	1.7	....	3.5	3.2	2.6	2.5
11	0.5	....	....	....	1.1	....	2.5	2.0	1.6	1.5
12	....	....	....	....	0.9	....	2.0	1.1	0.8	0.5
13	....	....	....	....	0.6	....	1.6	0.6	0.3	....
14	....	....	....	....	0.5	....	1.2	....	....	....
15	....	....	....	....	0.2	....	1.0	....	....	....
16	....	....	....	....	....	....	0.7	....	....	....
17	....	....	....	....	....	....	0.5	....	....	....
18	....	....	....	....	....	....	0.3	....	....	....
19	....	....	....	....	....	....	0.2	....	....	....
20	....	....	....	....	....	....	0.1	....	....	....

TABLE 2.—(*Continued*)

Interval No.	Stream Numbers:									
	1	2	3	4	5	6	7	8	9	10
	(c) COPPER BASIN WATER-SHEDS									
T^* =	7.5‡	15.0	2.5	2.5	2.5	2.5	....	....	....	....
1	0.7	0.6	0.1	1.5	0.3	0.8	....	....	....	....
2	1.9	3.4	0.2	4.6	1.0	3.5	....	....	....	....
3	5.6	7.8	0.6	27.4	2.6	16.5	....	....	....	....
....	....	....	....	(37.5)†	....	....	....	....	....	....
4	9.4	14.0	8.0	12.9	4.1	40.0	....	....	....	....
....	....	....	....	....	....	(56.0)†	....	....	....	....
5	15.6	19.2	24.0	10.0	6.3	19.8	....	....	....	....
....	....	(20.2)†	(34.8)†	....	....	....	....	....	....	....
6	20.3	16.0	24.0	8.7	8.9	13.7	....	....	....	....
....	(21.0)†	....	....	....	....	....	....	....	....	....
7	14.0	11.6	10.1	7.4	11.75	3.8	....	....	....	....
8	8.7	8.2	7.4	6.1	15.25	1.1	....	....	....	....
9	5.7	6.2	6.2	4.8	18.6	0.5	....	....	....	....
....	....	....	....	....	(20.0)†	....	....	....	....	....
10	4.4	4.4	5.2	3.6	16.1	0.2	....	....	....	....
11	3.7	3.2	4.0	2.8	11.55	0.1	....	....	....	....
12	3.0	2.2	3.1	2.5	2.8	....	....	....	....	....
13	2.3	1.4	2.5	1.7	0.5	....	....	....	....	....
14	1.7	1.0	1.5	1.2	0.2	....	....	....	....	....
15	1.4	0.6	0.9	1.0	0.1	....	....	....	....	....
16	0.9	0.2	0.5	0.9	0.05	....	....	....	....	....
17	0.5	....	0.4	0.8	....	....	....	....	....	....
18	0.2	....	0.35	0.7	....	....	....	....	....	....
19	....	....	0.3	0.6	....	....	....	....	....	....
20	....	....	0.25	0.4	....	....	....	....	....	....
21	....	....	0.2	0.2	....	....	....	....	....	....
22	....	....	0.1	....	....	....	....	....	....	....
23	....	....	0.05	....	....	....	....	....	....	....

* T = time interval, in minutes. † Peak percentages are shown in parentheses.
‡ See Fig. 4. § See Fig. 3.

bution graph was then chosen for each case. This was done either by selecting one of the individual graphs as representing an average, or by drawing an average graph through the cluster and taking the percentages from it. The latter process involves a trial-and-error procedure in order to produce a total of 100 per cent. Numerical values of the composite distribution graphs are listed in Table 2.

Characteristics of the Distribution Graphs.—An examination of the twenty-two sets of distribution graphs similar to Figs. 3 and 4 showed that there was a close agreement among the graphs in each group. Furthermore, for any stream, the shapes of the graphs were singularly individualistic. The adherence to the characteristic shape for each of the streams appears to be much closer than in the case of streams draining larger water-sheds.

The rainfall varied considerably within the individual clusters of graphs. For example (see Table 3), unit hydrographs selected for Coweeta 9 were produced by rains varying from 0.51 in. to 2.72 in., those of Coweeta 5 by rains varying from 0.35 in. to 1.69 in., and those of Bent Creek 1 varied from 0.51 in. to 1.44 in.

The actual peaks of the unit hydrographs also varied widely within the individual cluster. For instance, the peak discharges varied from 9 to 95 cu ft

per sec in the unit hydrographs used for Coweeta 9 and, in the case of Bent Creek 3, the peak discharge varied from 1.9 to 13.0 cu ft per sec.

The duration of each of the rains used in preparing the distribution graphs and the period of rise for each stream are given in Table 3. The period of rise corresponds approximately to the time of concentration in the so-called rational formula. The latter is usually defined as the time required after the beginning of rainfall for run-off to reach the point of concentration from the most distant point on the water-shed. These studies indicate that a somewhat different conception must be given to this time interval.

A careful study of Table 3 in conjunction with the corresponding distribution graphs will show that the peak discharge appears at approximately the same time after surface run-off starts, regardless of the duration or magnitude of the rainfall. Nearly every set of graphs provides illustrations of this. For

TABLE 3.—DESCRIPTION OF THE RAINS THAT PRODUCED DISTRIBUTION GRAPHS SIMILAR TO FIGURES 3 AND 4

Date of storm* (1)	Total rainfall, in inches (2)	Duration of rainfall, in minutes (3)
(a) BENT CREEK WATER-SHED		
Stream No. 1; P = 22 min:		
August 22, 1935	1.22	....
July 1, 1936	0.75	20
July 2, 1936	1.44	30
July 20, 1936	0.51	20
September 20, 1936	0.95	20
Stream No. 2; P = 15 min:		
July 20, 1936	0.50	20
August 6, 1936	0.58	20
August 27, 1936	0.81	20
September 20, 1936	0.37	15
September 29, 1936	0.66	30
Stream No. 3; P = 12 min:		
July 2, 1936	1.03	20
July 6, 1936	0.64	15
July 23, 1936	0.41	30
August 10, 1936	1.13	20
Stream No. 4; P = 45 min:		
August 29, 1936	0.71	25
September 20, 1936	0.44	15
September 29, 1936	0.69	30
Stream No. 5; P = 30 min:		
July 2, 1936	2.04	30
September 20, 1936	1.02	70
Stream No. 6; P = 12 min:		
August 23, 1935	1.38	50
July 2, 1936	2.22	30
September 20, 1936	0.85	70
Stream No. 8; P = 20 min:		
June 7, 1936	0.74	20
July 1, 1936	0.53	20
July 6, 1936	0.64	15
July 20, 1936	0.63	20
(b) COPPER BASIN WATER-SHED		
Stream No. 1; P = 38 min:		
June 8, 1935	0.69	25
June 21, 1935	1.51	55
July 5, 1935	0.80	30
May 12, 1936	1.94	60
Stream No. 2; P = 60 min:		
June 8, 1935	0.73	25
June 21, 1936	1.48	55
July 20, 1936	0.67	25
August 10, 1936	0.72	35
Stream No. 3; P = 10 min:		
April 2, 1935	0.35	8
July 2, 1935	0.90	7
March 24, 1936	0.74	8
June 10, 1936	0.89	12
August 10, 1936	0.70	16
Stream No. 4; P = 8 min:		
March 25, 1935	0.77	8
July 27, 1935	0.35	10
August 17, 1935	0.63	8
August 23, 1935	1.41	40
October 28, 1935	1.10	12
Stream No. 5; P = 20 min:		
April 2, 1935	0.29	8
April 3, 1935	0.73	15
July 29, 1935	0.67	8
August 4, 1935	0.28	14
Stream No. 6; P = 10 min:		
July 13, 1935	0.25	10
July 29, 1935	0.67	10
August 3, 1935	0.28	14
August 17, 1935	0.50	12

TABLE 3.—(*Continued*)

Date of storm* (1)	Total rainfall, in inches (2)	Duration of rainfall, in minutes (3)	Date of storm* (1)	Total rainfall, in inches (2)	Duration of rainfall, in minutes (3)
(c) COWEETA WATER-SHED					
Stream No. 1; P = 45 min:			Stream No. 7; P = 60 min:		
August 22, 1935	1.43	30	May 11, 1936	1.51	60
August 23, 1935	0.33	35	June 10, 1936	0.79	30
July 12, 1936	1.74	40	June 12, 1936	0.92	22
July 17, 1936	0.68	20	July 22, 1936	1.18	65
August 19, 1936	0.97	20	Stream No. 8; P = 70 min:		
Stream No. 2; P = 60 min:			May 11, 1935	1.36	120
July 12, 1936	1.94	40	July 25, 1935	1.15	65
August 19, 1936	1.02	20	July 26, 1935	1.37	35
August 24, 1936	0.99	20	August 22, 1935	1.49	30
Stream No. 4; P = 25 min:			Stream No. 9; P = 75 min:		
August 11, 1934	1.40	....	April 24, 1935	0.99	42
May 11, 1935	1.38	60	July 12, 1936	1.53	35
August 19, 1936	0.72	20	August 19, 1936	0.51	25
August 24, 1936	1.91	20	August 24, 1936	1.83	20
Stream No. 5; P = 15 min:			August 28, 1936	2.72	75
July 12, 1936	1.69	35	Stream No. 10; P = 60 min:		
July 13, 1936	0.35	16	June 4, 1936	1.13	15
August 19, 1936	1.09	25	June 12, 1936	1.06	22
August 24, 1936	1.60	42	August 24, 1936	1.17	20
Stream No. 6; P = 30 min:					
August 11, 1934	....	....	....	....	....
July 12, 1936	1.66	35	....	....	....
August 19, 1936	1.02	25	....	....	....
August 24, 1936	1.46	42	....	....	....

* P = Period of rise.

instance, the period of rise for Stream 9, Coweeta (see Fig. 3), is 75 min. Four of the distribution graphs were produced by rains lasting only from 20 to 42 min. These graphs fall very well in line with the one produced by a much heavier rain which fell during the entire 75 min. On Coweeta 7, rains lasting 30 and 22 min, respectively, produced distribution graphs which coincide with those produced by rains of 60- and 65-min duration. In contrast to these shorter rains, about ten of the rains used were of a longer duration than the period of rise, although, in most cases, the major portion of the rain fell within this period. However, even in the case of the rain of August 23, 1935, on Copper Basin 4 (which lasted five times as long as the period of rise) a definite peak occurred at the same time as those of the other distribution graphs. This evidence may be summarized as follows:

(1) Any rain occurring in a time equal to or less than the period of rise will produce a unit hydrograph;

(2) A rain that lasts somewhat longer than the period of rise but with decreasing intensity also produces a unit hydrograph; and,

(3) There is a tendency for the period of rise to be unaffected even if the rain continues longer at a high intensity.

The foregoing considerations help to trace the sequence of events that constitute the process of surface run-off. This process is believed to be as

follows: The first rainfall that occurs at a rate in excess of the infiltration capacity produces surface storage. As rainfall continues the surface water begins to move down the slopes in thin films and tiny streams. Surface tension and friction play an important part at this stage. As these minute rivulets increase in volume, their velocity becomes greater; but their path is tortuous and every small obstruction causes a delay until sufficient head is built up to overcome it. Upon its release, the stream is suddenly speeded on its way again. Each time that two or more of these little streams combine, the water is further accelerated in its down-hill path. It is the culmination of all these small contributions into a final combined accretion to the main stream which produces the ultimate hydrograph. It appears, therefore, that the period of rise has no particular relation to the time required for water to reach the gaging station from the most remote part of the water-shed, since the peak is merely the result of the simultaneous release of the accumulated rainfall from the major portion of the water-shed. The rain that falls near the end of the period begins its overland flow much more readily than that falling earlier, since it falls at a time when all the surface depressions have been filled and into water which is flowing with a relatively high velocity. An increase in the intensity of the rainfall over the minimum required to produce surface run-off increases the total surface run-off but does not materially change the time required to complete the foregoing process.

Applications of the Distribution Graph

It has been shown that unit hydrographs exist in, and may be isolated from, the discharge records of small streams. A study of the natural phenomena involved in the formation of a unit hydrograph has thrown light on some phases of the process of surface run-off. In the following studies, the distribution graphs are utilized to test the value of the unit hydrograph principle in surface run-off analysis.

Correlation of the Distribution Graph with Water-Shed Characteristics.—The variety of types and sizes of water-sheds upon which the investigation is based offers an opportunity to relate the characteristics of the distribution graphs with those of the water-sheds. The writer is aware that the exact dimensions of the distribution graphs are partly a matter of personal judgment, especially the widths of bases and the peak percentages. A change in the method of separating base flow from storm flow might change, considerably, the total length of base of the distribution graphs, although it would not materially affect their general shape. Furthermore, in some of the streams, the records may not have been long enough to produce a true unit hydrograph; however, as demonstrated later by the construction of pluviagraphs, the evidence is in favor of their adequacy. In view of the foregoing consideration, the correlations were made to include as much information as possible. None of the graphs contained "average curves." The only averages used were the composite distribution graphs for each stream.

For a comparison of peak discharge, the twenty-two composite distribution graphs were reduced to a 2.5-min basis. The peaks were then plotted in order of magnitude as shown in Fig. 5.

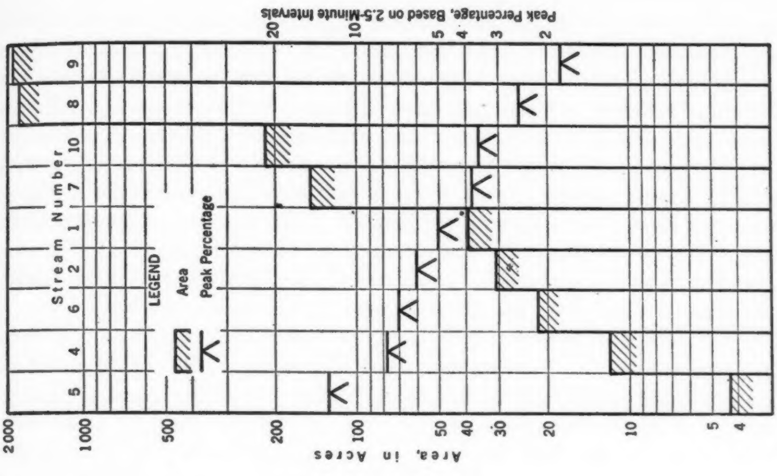


FIG. 6.—RELATION BETWEEN PEAKS OF DISTRIBUTION GRAPHS AND AREAS; COWEETA WATER-SHEDS

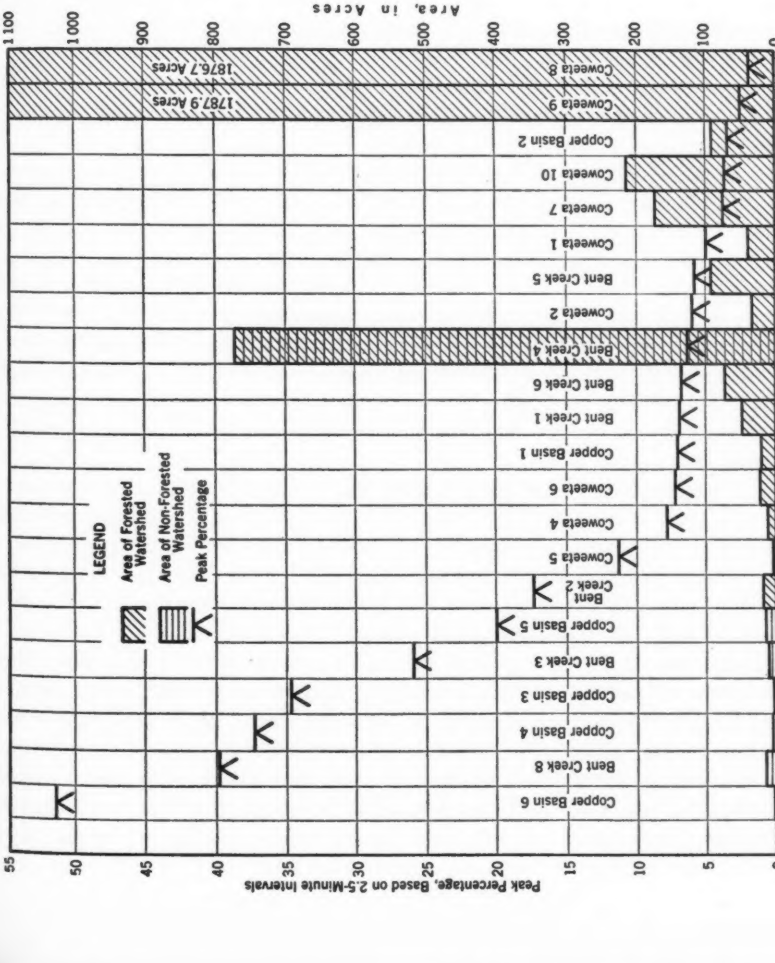


FIG. 5.—RELATION BETWEEN PEAKS OF DISTRIBUTION GRAPHS AND AREAS OF WATER-SHEDS

That the peak percentage varies according to the area of the water-shed is quite obvious. However, it is the manner in which the variation occurs that is especially interesting. For instance, if the forested water-sheds of the Bent Creek, Coweeta, and Copper Basin areas are considered separately, the agreement between peak percentage and area becomes consistent. That is, from left to right on the graph the areas are constantly increasing. This may be noted by a careful study of Fig. 5; but it is more clearly indicated in Fig. 6, in which only the Coweeta streams are plotted. The greatest peaks for a given area are found at Bent Creek—the smallest at Copper Basin and Coweeta. It is significant that the denuded and grassed areas have peaks much higher than would be indicated by the general trend of the forested areas.

It is interesting to consider, specifically, some of the individual variations from the general trend. For instance, Bent Creek 8 is considerably out of its place, considering merely area and peak. It has the largest area of any of the unforested water-sheds, but has the second highest peak. It is possible that this is due to the fact that this water-shed has nearly a semi-circular shape, with its outlet at the center of the circle, thus permitting all sectors of the water-shed to contribute their run-off simultaneously with practically no delay due to channel storage.

Water-shed 4 on Bent Creek has approximately the position in the graph that one would expect, being about 50% forested and 50% cultivated farmland, abandoned farmland, and cut-over land.

The peak percentage of Stream No. 2, Bent Creek, is considerably above the trend of the other forested water-sheds. Two-thirds of the standing trees were cut from this water-shed in 1930 and 1931, and skidding operations at that time were exceptionally heavy. Furthermore, drainage from approximately 100 yd of roadway discharged into the water-shed. Although the quantity of water entering from this source was not great, it entered in the form of a small stream and followed a fairly narrow and well-defined course to the main stream, discharging into it at a point about 100 ft from the weir. This latter difficulty was eliminated in subsequent records to determine whether the high peak was due to the scarcity of vegetative cover or to the unnatural drainage.

Fig. 7 was prepared to show the variation in the duration of run-off for a unit storm, as indicated by the lengths of bases of the distribution graphs. The bases are plotted using the area, in acres, as the zero ordinate for each graph. The types of water-sheds are indicated by the legend. As is to be expected, the length increases with the area. The sharp difference between the forested and unforested areas is apparent.

Although the individual variations from the general trend (with possible causes as stated previously) are interesting and perhaps significant, the important feature is that there appears to be a general relationship between the dimensions of the distribution graph and the area, and a logical reason for all the deviations. If this is true, the value of distribution graphs as a means of isolating the area factor in the analysis of surface run-off phenomena becomes apparent.

Instead of using a uniform interval of 2.5 min as a percentage basis in the foregoing analysis, it would have been possible, and might at first appear logical, to base the percentage upon a constant portion, say 10%, of the base of the distribution graphs. The effect of selecting the proper length of base lines becomes much more pronounced when this method is used. For instance, the peak percentages based on 2.5-min intervals are 4.0% for Base Line *a* and

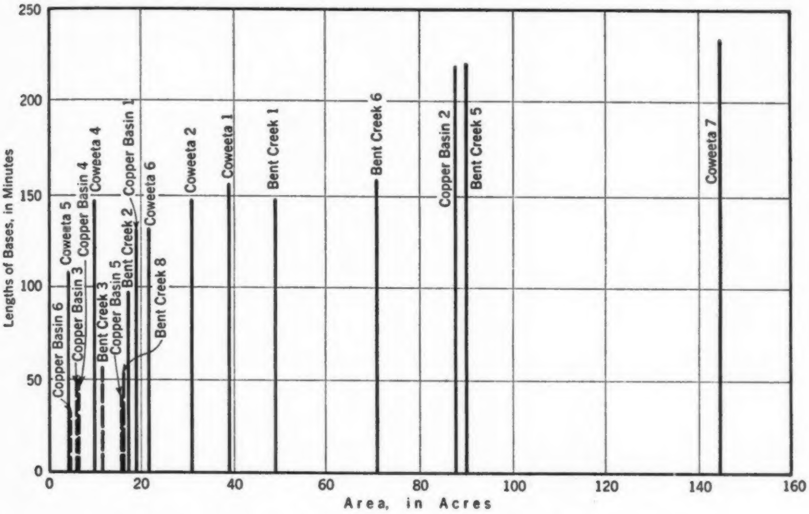


FIG. 7.—RELATION OF WATER-SHED AREAS TO BASES OF DISTRIBUTION GRAPHS

4.4% for Base Line *b*, Fig. 2(a). Based on an interval equal to 10% of the length of bases, the peak percentages are, respectively, 43.6% and 31.9 per cent. The difference in the first case is 10% and in the latter case about 35% showing that personal judgment in selecting the base line would play a much more important part if the latter method were adopted.

For this reason, and because the peak percentage based on a definite time interval may be translated into rate of surface run-off and therefore has a real practical value, the comparisons were all based on such intervals.

Application of the Pluviagraph.—The pluviagraph is a hydrograph of 100% surface run-off prepared by distributing a rain or a series of rains according to the ordinates of the distribution graphs. It is assumed that each unit storm produces a unit hydrograph, although other subsequent or succeeding rains may produce overlapping graphs. Then, since a distribution graph shows the successive periodic surface run-off contribution of a unit hydrograph, each unit storm must contribute to each time interval, after the beginning of rainfall, a portion determined by the successive percentages of the distribution graph. Only that rainfall which falls at a rate greater than the infiltration capacity becomes surface run-off. Therefore, a coefficient must be applied to the pluviagraph values to produce an actual hydrograph of surface run-off.

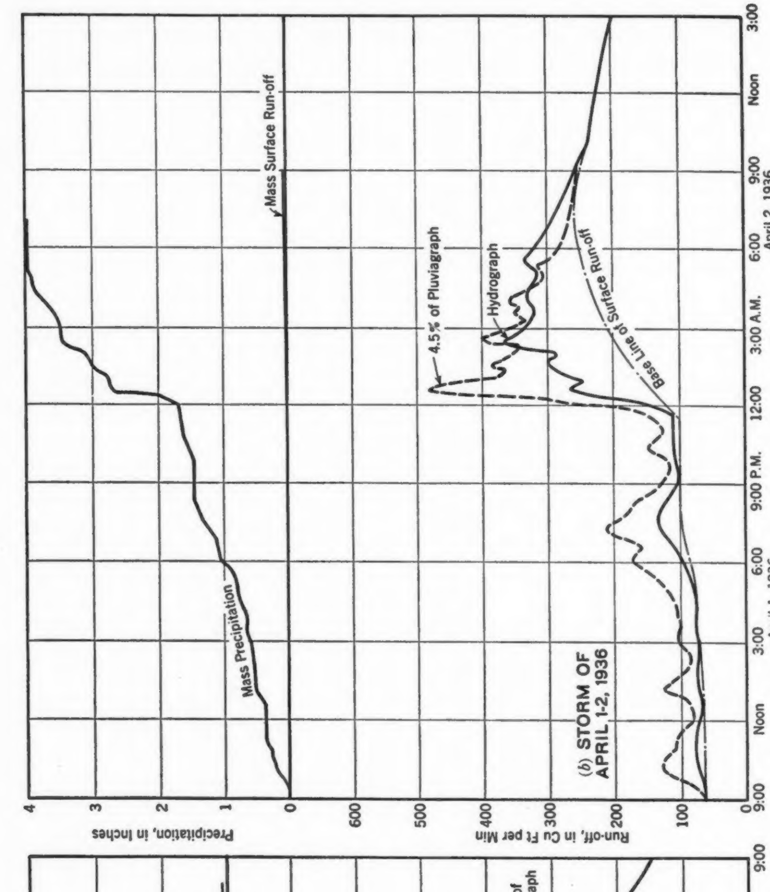


FIG. 8.—PLUVIAGRAPHS FOR TWO STORMS; STREAM NO. 7, COWEETA

The preparation of pluviagraphs has been adequately described elsewhere.[6] The selection of the time interval into which the distribution graphs are divided must be made arbitrarily. Lengthening the interval shortens the labor involved in computing the pluviagraph. The length is limited, however, to such a unit of time as will give a smooth and correct curve. For example, it was found that a 30-min interval was too long to be applied to Stream No. 2, Copper Basin, with a water-shed area of 89 acres, whereas a 15-min interval produced satisfactory results; 15-min intervals were also used for Stream No. 7, Coweeta, which has a water-shed area of 145 acres. In the case of Stream No. 3, Bent Creek, which drains an area of 10.8 acres, 5-min intervals were used.

The division of the continuous precipitation into individual rains or unit storms requires considerable care and usually some experimentation. In general, the rain was divided according to changes in intensity. However, if any rate of rainfall continued for too great a time, it was found necessary to break it up arbitrarily into parts, especially if the rain was of low intensity. In the case of Copper Basin 2, and Coweeta 7, a half-hour was found to be an adequate unit of time into which to break these rains. However, on Stream No. 3, Bent Creek, much of the rain was broken down into 5-min intervals. It was found that the procedure became clarified after several trials on each stream. The rainfall records must be of such a type that the rain can be broken down, easily and accurately, into less than 5-min periods in order to apply this method successfully to very small water-sheds.

Fig. 8 shows two pluviagraphs prepared for Stream No. 7, Coweeta, from which it may be seen that the actual hydrograph is quite faithfully reproduced. These pluviagraphs would seem to be conclusive evidence that the unit hydrograph principle may be applied to small water-sheds.

Run-Off Coefficients and Infiltration Capacity.—In every case where a pluviagraph was prepared for a relatively intense series of rains, during which surface run-off was continuous, it was found that the percentage of the rainfall discharged as surface run-off became progressively greater during the course of the rain. This may be observed in Fig. 8. The values plotted are the pluviagraph data reduced by some constant percentage. The percentage chosen for each graph was such as to bring the pluviagraph into the region of the actual hydrograph. The percentage value used is always too great at the beginning; it becomes correct sometime during the course of the storm; and the value is too small near the end of the storm. It is indicated, therefore, that the infiltration capacity decreased during the course of the rain. Graphs such as these suggest that the pluviagraph provides a means of tracing the quantitative changes in infiltration capacity, or run-off coefficient, during continuous rainfall, thus permitting a correlation of these quantities with rainfall characteristics.

The correlations attempted in the following pages are based on quantity of rainfall and do not specifically take rainfall intensity into account. Although it is recognized that intensity of rainfall has an important bearing on the percentage that becomes surface run-off, the intense rains are usually large ones, and therefore, if the run-off coefficient or the infiltration capacity is

correlated with quantity of rainfall, the intensity will be taken into account somewhat. There are several reasons for leaving intensity "out of the picture." In the first place, the determination of mean rainfall intensity for a given series of rains often leads to a distorted result, dependent upon how much of the rain is taken into account. Furthermore, it is difficult to include antecedent rainfall in a correlation involving intensity rather than quantity of rainfall.

In order to determine, quantitatively, the variations in percentage run-off, the percentage which brought the pluviagraph into practical coincidence with the actual hydrograph was determined for each significant peak during the course of surface run-off. This was then related to the quantity of rainfall, which, according to the pluviagraph, had had an opportunity to be converted to run-off at the time when that coefficient was applicable. In Fig. 9, each group of connected points represents such a series of consecutive rains during which surface run-off was continuous.

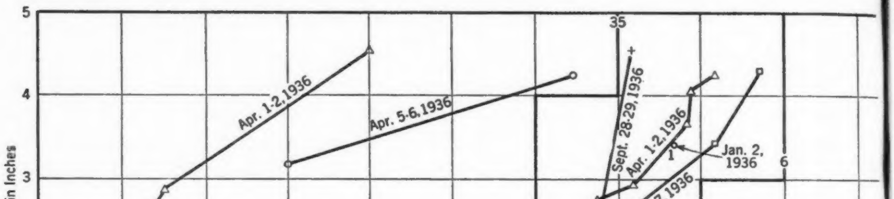


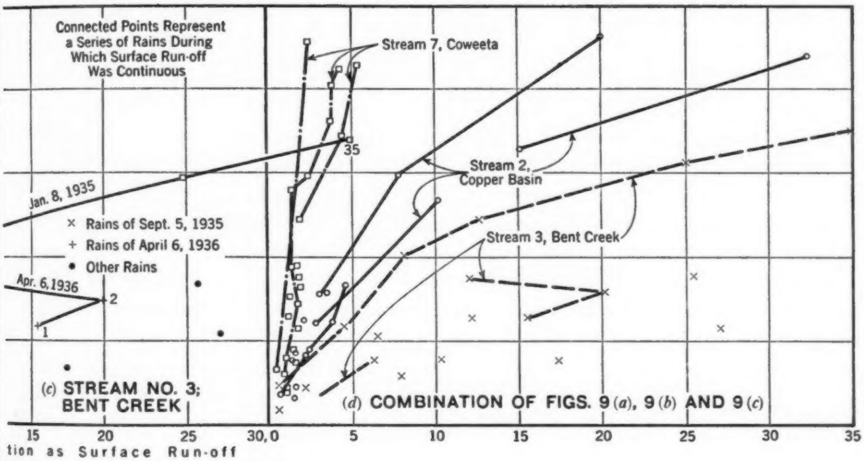
FIG. 9.—RELATION BETWEEN ACCUMULATE

Four of such series are plotted for Stream No. 2, Copper Basin, in Fig. 9(*a*), together with a number of points from individual rains. It will be seen that there is a definite relationship between accumulated rainfall and percentage of surface run-off, despite the fact that rain intensity and previous saturation are not considered. It will be noted that points plotted for the storm of April 5 and 6, 1936, have higher percentages than for the preceding storm of April 1 and 2, 1936. Undoubtedly, the first storm had an appreciable effect on the second one, and, if this could be taken into account, the two curves would probably come near to coinciding. As the study of soil moisture and its effect upon surface run-off is advanced, this may become possible, but for the present it is more important to know the range within which the coefficients will vary.

Fig. 9(*b*) is a similar graph for Stream No. 7, Coweeta. It will be seen that the trend is noticeably different in this case. The effect of previous

saturation noted for the two April storms may be seen again for the same storms on this stream.

Three sets of coefficients in series, and a number of single ones, are plotted for Bent Creek 3, in Fig. 9(*c*). The series of storms occurring on January 7, 1935, is a good example of the rising percentage of surface run-off. At the time Rain No. 1 occurred, 1.72 in. of rainfall had fallen. Since surface run-off was intermittent to this point, the previous rainfall was not considered in plotting the points. The intensity of rainfall was relatively low throughout this rain. The fact that these points lie in a lower percentage range than the main trend would indicate that intensities of rainfall have an important effect in water-sheds of this nature. More evidence of this is found in the series of rains occurring on April 6, 1936. Rain No. 3 of this series was a much less intense shower than Nos. 1 and 2. It may be seen from the graphs that the percentage of surface run-off has dropped back for this rain.



RAINFALL AND PERCENTAGE RUN-OFF

Thus far, no mention has been made of the length of time in which a build-up in the coefficient is sustained after the end of surface run-off. In this connection it is interesting to consider Rain No. 4 of the series just mentioned. In Fig. 9(*c*), Point 4*b* is plotted according to accumulated rainfall, whereas Point 4*a* is plotted according to the rainfall that produced its run-off. A value lying between these two would fall in line with the general trend. A break in the continuity of surface run-off of 1 hr occurred between this rain and Rain No. 3. Although Rain No. 4 was nearly equal in amount and intensity to Rain No. 3, the percentage of surface run-off is much smaller. Surface detention may account for some of this difference, but the following considerations indicate that its effect could not have been great: (1) The slope is relatively great and erosion has advanced to such a stage on this water-shed that actual detention in the form of puddles on the ground must be very slight; and (2) in the several hours intervening between the rains, the relative humidity

remained at nearly 100%, so that practically no evaporation occurred. Therefore, the quantity of rainfall previously intercepted and held by the vegetation had not been depleted, and detention from this source may be practically eliminated. The conclusion may be drawn, then, that the infiltration capacity increased considerably in the space of 1 hr despite the 1.60 in. of rainfall immediately preceding.

Another illustration is furnished by the rains of September 5, 1936. Surface run-off resulting from these rains was interrupted for periods varying from a half to 1.5 hr between all of the rains except Nos. 4 and 5. For illustration, Point 6*b* was plotted according to accumulated rainfall whereas Point 6*a* was plotted according to the actual rainfall. Since Point 6*a* falls into line with the other points, it appears that the effect upon infiltration capacity of the previous 1.10 in. of rainfall is completely gone in 1.5 hr.

Five heavy rains occurring in the period from April 2 to April 9, 1936, on Coweeta 7 are plotted individually in Fig. 9(*b*). Breaks in the occurrence of surface run-off between the successive storms were 29, 28, 32, and 13 hr in duration, respectively. The amount of each rain is shown by the graph. It may be seen that the points plotted individually fall quite well in line with the general trend, which indicates that these rather heavy rains had little or no effect on the surface run-off coefficient of those following.

The points plotted for each stream in Figs. 9(*a*), 9(*b*), and 9(*c*) are all combined in Fig. 9(*d*). The definitely distinct trends of the points for the three streams are evident, despite the limitations in allowing for intensity of rainfall and previous saturation. If such a series were prepared for a sufficient number of streams, covering a wide enough range in area, cover type, slope, etc., the resulting information would be of value to the practicing engineer, as well as being an important step in the accumulation of a basic knowledge of surface run-off phenomena. It would give the engineer definite limits between which the coefficient could be expected to vary for a given type of water-shed. If he were then able to choose a representative distribution graph, he could predict, far more accurately than he could by the use of any empirical formula, the run-off to be expected from a given quantity of rainfall.

It will be noted that no attempt was made to separate winter and summer rains in the foregoing correlations. A similar study on northern streams would undoubtedly require such a separation. Furthermore, although the indications are that the effect of previous saturation on infiltration capacity is negligible on these water-sheds, it is likely that in other locations this effect may be extremely important.

The value of a series of investigations to determine run-off coefficients under varying conditions cannot be over-estimated. It is extremely desirable that data collected from different sections of the United States be gathered and analyzed to show the effect of soil structure, topography, vegetative cover, temperature, and other water-shed and rainfall factors on the surface run-off coefficient or infiltration capacity.

The Use of the Unit Hydrograph in Estimating Flood Flows

The design of so many structures depends upon a knowledge of the maximum rate of run-off to be expected from a water-shed that any method that will give a more dependable determination of this quantity must be of inestimable value to the engineering profession. The unit hydrograph method possesses several advantages over other methods usually employed for this purpose. First, this method permits the engineer to estimate not merely the peak discharge, but the entire hydrograph of run-off. Second, the peak may be determined for any desired time interval. It thus provides a flexibility that is not provided by any empirical formula. Third, the quantities involved in computation by this method have physical conceptions which are familiar to the engineer. No empirical formulas with vaguely defined constants are involved; the physical characteristics of the water-shed have a straight-forward bearing on the computations. Finally, the method lends itself easily to the experimental determination of the quantities involved—namely, percentage of rainfall appearing as surface run-off and the shape of the distribution graph.

An outline of the procedure that can be followed in estimating the flood flow from a water-shed for which no records are available follows:

(1) A complete knowledge of the physical properties of the water-shed must be secured. Specifically, the size, shape, topography, soil type, and nature of the vegetative cover must be determined. In areas where the underlying geologic structure may vary radically from place to place, it is important to determine the depth and orientation of impervious strata.

(2) Collect all the available stream-flow, rainfall, and water-shed data from similar drainage basins, preferably from the same region.

(*a*) Study the physical characteristics of these water-sheds in relation to the shapes of their distribution graphs. Correlations in the nature of those illustrated in Figs. 5, 6, and 7 would reveal the desired information. Determine the position of the water-shed in question with respect to those used in the correlation as defined by the physical properties of the water-shed. The shape of its distribution graph is thus also defined within reasonable limits.

(*b*) Determine the range of variation of run-off coefficients. The accumulation of all the available data in graphical form as illustrated in Fig. 9(*d*) would show the variations of this coefficient with rainfall and would permit the selection of a maximum value.

(3) Determine the maximum storm that has occurred in the region which might reasonably be expected to occur on the water-shed being studied. Transpose this rainfall upon the water-shed and compute the volume of water applied to the water-shed.

(4) The portion of this volume that will appear as surface run-off may be computed by applying the coefficient determined in Item (2*b*). The hydrograph of surface run-off may now be constructed by apportioning this quantity according to the figures of the distribution graph selected in Item (2*a*).

Where flood estimates are to be made on a water-shed for which some records are available, the results may be obtained in a similar manner except that the records on the water-shed itself would be utilized to the fullest extent in the foregoing procedure.

It is evident that the determination of the maximum flood becomes more dependable as the number of available stream-flow and rainfall records increases. A great mass of such data has been collected in the United States, but its usefulness is limited until it is brought together and analyzed in a systematic manner. Furthermore, for very small water-sheds, average daily discharge records of a stream are of little value for any type of stream-flow analysis. It is necessary that records on such streams (and likewise the rainfall records) be continuous, and that they be available as such to permit their utilization in the advancement of the knowledge of stream behavior.

Summary

The preparation of unit hydrographs and distribution graphs for twenty-two small streams and a study of their relation to the corresponding rainfall and water-shed characteristics have served to reveal some of the natural phenomena involved in the process of surface run-off as well as to point out the usefulness of the unit hydrograph principle in flood-flow prediction. The results are summarized as follows:

(1) Unit hydrographs are produced on small streams and may be readily selected from continuous run-off records;

(2) The ground-water contribution may be separated from the surface run-off with considerable assurance in the case of hydrographs resulting from isolated rains;

(3) The unit hydrograph is an important aid in the separation of ground-water contribution from surface run-off in the case of a complex hydrograph produced by a series of rains;

(4) Any rain that is sufficiently intense to produce surface run-off will produce a unit hydrograph if its duration is equal to, or less than, the period of rise;

(5) The period of rise is a function of the water-shed characteristics and is the time required for the major portion of the water-shed to release its accumulated rainfall load;

(6) There is a definite correlation between the shape of the distribution graphs and the water-shed characteristics;

(7) The reproduction of known flood hydrographs by the use of the pluviagraph provides a method of determining the variation of run-off coefficient or infiltration capacity on a water-shed throughout the period of surface run-off; and,

(8) The unit hydrograph principle provides a fundamentally sound method of predicting surface run-off.

The study has indicated certain lines of investigation that should be conducted to strengthen some features of the unit hydrograph principle and

to provide a background of experimental data upon which to base its application in flood-flow prediction. The studies which it is believed would be most fruitful are:

(a) An intensive examination of the variations in stream levels and adjacent water-table levels during the course of surface run-off to provide a dependable basis for separating ground-water flow from surface run-off. An exhaustive study on only a few water-sheds would provide data upon which to base the separation in other cases.

(b) A study to determine whether a composite or average depletion curve may be used to estimate a hydrograph of run-off during intervals between rains.

(c) An investigation to determine the influence of rainfall and water-shed characteristics upon surface run-off coefficients. (Such a study should attempt to isolate and show the effect of various factors such as size, shape, and slope of the water-shed, vegetative cover, cultivation, air temperature, duration and intensity of rainfall, antecedent rainfall, etc. The results would provide a basis for estimating the coefficients in cases where no discharge or rainfall records are available, and would indicate how changes on the surface of the water-shed would influence run-off.)

(d) The preparation and systematic analysis of distribution graphs from a large number of streams showing variations in the shapes of the graphs with relation to water-shed characteristics. The knowledge provided by such an investigation in conjunction with a known surface run-off coefficient would permit the construction of flood hydrographs on streams where no discharge records are available.

Acknowledgments

In its original form, this paper was a thesis presented to the University of Michigan, in 1937, in partial fulfillment of the requirements for the degree of Doctor of Philosophy. The work was first revised under the direction of members of the Department of Civil Engineering, with the special aim of making it a useful contribution to engineering literature. The study is based on unpublished records of rainfall and stream flow gathered by the Appalachian Forest Experiment Station, U. S. Forest Service, with which the writer was formerly associated.

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PAPERS

DEVELOPMENT OF THE COLORADO RIVER IN THE UPPER BASIN

By Thomas C. Adams,[1] M. Am. Soc. C. E.

Synopsis

In a general way, this paper treats the past and prospective use of water of the Colorado River and its tributaries in the upper basin above Lee's Ferry, Ariz. Descriptions are given on the topographic, industrial, and agricultural features connected with the basin and the effect of the Colorado River Compact and the Boulder Canyon Project Act on the use of water. Water supply for the upper basin, after agreements between the States concerned are satisfied, is shown to be somewhat less than had been anticipated by some.

Conditions favoring and limiting future extension of irrigation in the upper basin are described. Additional irrigation projects of the normal type will be built; other projects, primarily to aid the livestock industry of the region by the irrigation of meadows and feed crops, are held to be desirable; and, the large diversion projects conveying water outside the water-shed of the Colorado River for irrigation, power, industrial, and municipal purposes are shown to be highly important in the general plan of upper-basin development. The hydro-power resources, the rôle to be taken by power generation in water development, and the relations between the use of water of the upper basin and the social, economic, and industrial growth of the basin and near-by regions, are discussed. The paper includes an appendix containing data on the irrigable areas of the upper basin and an analysis of the flow of the Colorado River at Lee's Ferry (where division is made between upper and lower basins) since the signing of the Compact, which is believed particularly significant as it handles flow during the period of lowest known river flow.

Topography and Industry

Lee's Ferry is situated on the Colorado River of the West, in Arizona, immediately below the State line between Utah and Arizona (see Fig. 1). The

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area drained by this river and its tributaries up stream from the old ferry site is known as the upper basin. It is an extensive shallow basin "funnelling" to its river outlet, rimmed on the east by the crest of the Rocky Mountain Cordilleran and to the west by the somewhat lower crests of the Wasatch Mountains and those other mountains and high plateaus to the north and south. The basin comprises the highest area of so large proportions in the United States. It includes within its water-shed a varied topography of well-watered mountain heights, delightfully green mountain valleys and stream canyons, flat mesas, barren deserts, and profound box canyons. Adjacent to the basin, and just over the mountain crests on both the east and the west sides, are areas somewhat lower in elevation, having rich soil and more equable climate, and containing some of the most populous agricultural and industrial areas, of the western interior.

Localities within the upper basin and near-by are noted for their mineral wealth, and the region will continue for a long time to produce large quantities of non-ferrous and ferrous metals, coal, oil, natural gas, and a large variety of other non-metallic and rare minerals. The principal industries are stock raising, farming, mining and treating minerals, and the distribution of commodities produced elsewhere. The upper basin of the Colorado River is 104 000 sq miles in area and lies partly in each of the States of Utah, Colorado, Wyoming, and New Mexico. It is one twenty-ninth the area of continental United States and hence nearly twice the area of an average State. It is well supplied with railroads and highways but is only sparsely populated.

The mountain rims of the upper basin and other mountains lying within it receive a high rate of precipitation and from this small part of its area flows nearly 80% of the entire water supply of the Colorado River Basin, one thirteenth of the area of the United States. These mountains support no farming or agriculture other than open grazing because of the extremely short growing season. All other lands in the basin, with the exception of limited acreages of "dry-farm" wheat fields, are deficient in rainfall and must be irrigated to produce crops. Much of the area of the upper basin lies at the foot of the mountains and is itself high in elevation (more than 6 000 ft). It comprises broad plains, valleys, and mesas, the growing season of which is short. Agriculture is limited largely to the raising of livestock and the irrigation of feed crops. Lower parts of the upper basin, in southeastern Utah and adjacent parts of Colorado, have suffered deep erosion in recent geologic times and are a jumble of box canyons and desert mesas and plateaus which, in some places, are almost devoid of soil. Irrigated lands are limited to favorably disposed positions in stream valleys.

Irrigation Development

Irrigation development of the upper basin began early (William H. Ashley[2] and his trappers entered the Uinta Basin in 1824) and has proceeded in the manner typical of western irrigation. At the beginning small ditches were constructed by individuals and associations of individuals. These were fol-

[2] "History of Utah," by Bancroft.

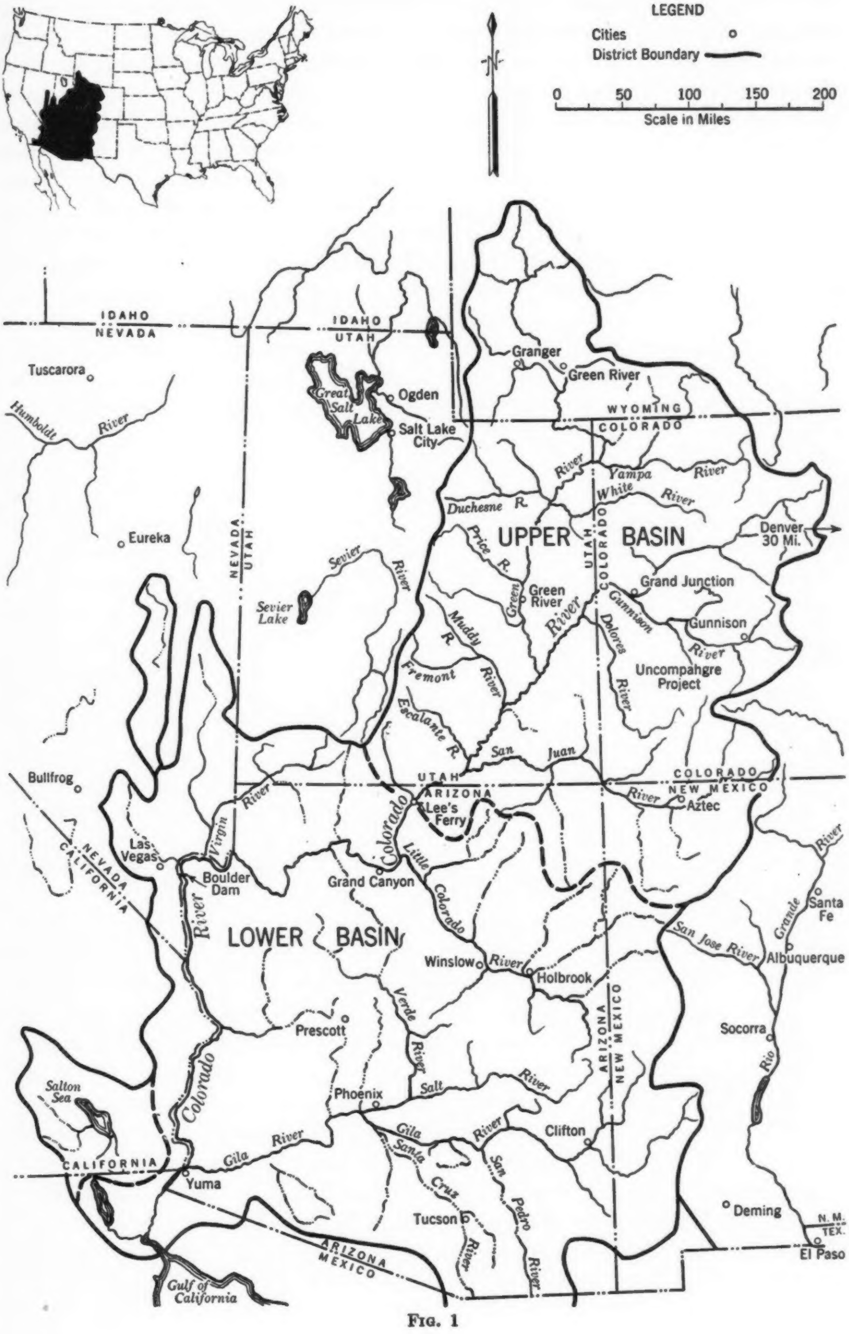
LEGEND
Cities
District Boundary
0 50 100 150 200
Scale in Miles
IDAHO
NEVADA
UTAH
WYOMING
COLORADO
ARIZONA
NEW MEXICO
CALIFORNIA
MEXICO
N. M.
TEX.
UPPER BASIN
LOWER BASIN
Tuscarora
Humboldt River
Eureka
Bullfrog
Great Salt Lake
Ogden
Salt Lake City
Sevier Lake
Sevier River
Granger
Green River
Yampa River
White River
Duchesne R.
Price R.
Green River
Denver 30 Mi.
Grand Junction
Gunnison
Gunnison River
Dolores River
Uncompahgre Project
Muddy R.
Fremont River
Escalante R.
San Juan River
Aztec
Lee's Ferry
Virgin River
Las Vegas
Boulder Dam
Grand Canyon
Little Colorado
Colorado River
Winslow
Holbrook
Santa Fe
San Jose River
Albuquerque
Rio Grande River
Socorra
Prescott
Verde River
Salt River
Phoenix
Salton Sea
Gila River
Clifton
Santa Cruz River
San Pedro River
Yuma
Tucson
Deming
El Paso
Gulf of California

FIG. 1

lowed by larger projects built by companies and, co-operatively, by users; and, finally, it was followed by those constructed by the United States Bureau of Reclamation under increasingly lenient financial terms. Development has proceeded steadily, but in recent decades developers have encountered the need for more thorough planning of projected irrigation works, the necessity of dealing with projects of larger scale and complication, of being satisfied with serving land less advantageously situated with respect to the water supplies, and using dams sites of higher unit cost. Furthermore, as time has progressed, there has come a realization that the few million acre-feet of water coursing down the streams of the basin are a vital economic factor of the region and, because of their meagerness, must be carefully conserved.

Agricultural production of the upper basin is generally not competitive with that elsewhere in the United States. It is small in comparison—much of it is consumed locally, and the remainder is mainly supplementary to national agriculture as represented by the livestock and the specialized irrigated crops.

Before 1922 a demand for the use of more water of the Colorado River in the lower basin and for the construction of a large dam and reservoir to permit this use and to provide flood protection for the irrigated valleys of the Colorado Delta and electricity and water for Los Angeles, Calif., led to negotiations between representatives of all States of the basin which culminated in the Colorado River Compact signed in November, 1922. The Compact was supplemented and approved conditionally by the Boulder Canyon Project Act of December, 1928. Under the terms of the Act the Compact became effective as regards all States concerned, except Arizona (which refused to ratify the compact) in June, 1929, six months after the Act was signed. California was required, by the Act, to accept the provision that no more than 4 200 000 acre-ft, plus one-half the surplus which might later be allotted to the lower basin, would be consumed in the State. The Compact reserved for the upper basin consumptive use of 7 500 000 acre-ft of water per yr but subjected this to an obligation which takes precedence; namely, that in any 10-yr period a total of 75 000 000 acre-ft of water are to be permitted to flow from the upper basin past Lee's Ferry gaging station for the benefit of the lower basin. Further provisions are that no water "which cannot reasonably be applied to domestic and agricultural uses" is to be withheld in the upper basin and that any water which may be granted to Mexico shall be supplied first from the surplus flow of the river in excess of the amounts allotted to the upper and lower basins; or, in the event that this surplus is insufficient to meet the Mexican grant, the deficiency is to be supplied half from each of the upper and lower basin allotments. The Compact is a comparatively simple document intended to permit lower-basin developments to proceed and to preserve sufficient water for the upper basin developments which are intended to follow in due time. It "sets aside" questions of priority of appropriation as between the upper and lower basins. Assuming good faith by all parties, the Compact may well accomplish its objects and be beneficial to every one. It is an equitable document.

Under the Boulder Canyon Project Act no work was undertaken; nor was any immediate benefit in the nature of construction intended, to the upper basin, with the exception of a moderate appropriation for studies of proposed developments throughout the entire basin. The Act set aside receipts from the use of works to be constructed under it after repayment to the Government of all costs of these is completed, in a fund which presumably would be devoted, at the direction of Congress, to further construction in the basin and partly in the upper basin. Through the construction of the storage reservoir at Hoover Dam, the Act was intended to remove the possibility that litigation in support of prior appropriations made of Colorado River water in the lower basin would interfere with, or prevent, further developments in the upper basin.

Appraisal of Past Experience

Sixteen years have now (1939) elapsed since the signing of the Compact and ten years since the passage of the Boulder Canyon Project Act; and it is of importance to upper-basin development to know what light and fruition intervening years have brought to the terms and intentions of these notable documents. Since 1922 many more discharge measurements of the Colorado River have been made and earlier discharge records corrected. Furthermore, the most severe drought period known has occurred, from which recovery is being made only during the present year. A review of the discharge of the Colorado River at Lee's Ferry since 1922 discloses an average measured discharge of 12 750 000 acre-ft per yr. To this may be added an average estimated upper-basin depletion of 2 180 000 acre-ft per yr, totaling 14 930 000 acre-ft per yr, which is an amount on the average for the seventeen years, 1922–1939, slightly less than sufficient to meet the Compact provision of 7 500 000 acre-ft per yr to each of the two divisions of the river basin. These values may be compared with an average, all-time, reconstructed flow of about 17 000 000 acre-ft per yr believed to be the yield of the river system above Lee's Ferry when the Compact was drawn. The low average discharge since 1922 would have the effect of reducing the all-time average to date by a small amount. For the last ten years of record (1929–1939), the values corresponding to those given are 11 720 000, 2 110 000, and 13 830 000 acre-ft per yr, respectively. To obtain the total quantity of water available for use from the Colorado River and its tributaries in both the upper and lower basins, after the terms of the Compact are applied, adjustments for gains and losses in the lower river must be made and a deduction taken for water granted to Mexico. No statement can be given of water that may be granted to Mexico. Only that title to water there established by treaty need be recognized and no treaty exists. The present use of water from the Colorado River in Mexico is about 1 000 000 acre-ft per yr.

A further study of the Lee's Ferry discharge shows that at the end of the water year, September 30, 1937, there would have been a deficiency in the 10-yr cumulative amounts (which the Compact specifies must be not less than 75 000 000 acre-ft for any 10-yr period) of 5.5 million acre-ft if use of water in the upper basin had reached the proportions requiring 7 500 000 acre-ft

per yr in a year of normal stream discharge (which amount would be appropriately less in drought years) and in the absence of large, long-carry-over storage in the upper basin available to supplement stream discharge. Furthermore, even if the discharge of the river increases and remains above average during the ensuing years beginning with 1939, the deficiency in the 10-yr cumulative discharge will increase to perhaps the end of the water year of 1939 when it will possibly be 10 000 000 acre-ft. It is not likely that this 10-yr amount would again reach the 75 000 000 limit until well into the 1940's, barring continuously and phenomenally high discharge of the river.

Other studies of discharge show that in the ten years just passed an average of 5 800 000 acre-ft per yr would be available for upper-basin use from direct stream flow had the upper basin been using all available to it under the Compact, with adherence to the 10-yr cumulative provision included but without being obliged to share in an allowance of water to Mexico (see Column (12), Table 2 in the Appendix). Considerable storage would be required on upper tributaries of the river to equalize this average flow, throughout the 10-yr period, in a manner acceptable for upper-basin use. Water available to the upper basin under the conditions specified could be increased appreciably if large carry-over storage on the tributaries or the main river in the upper basin was available to assist in supplying 10-yr cumulative demands of the lower basin at Lee's Ferry.

These studies also show that an average of 6 700 000 acre-ft per yr during the ten years, 1929–1939, would be available from direct stream flow at the outlet of Hoover Dam under the foregoing conditions (see Column (14), Table 2). The equalization of this flow could take place in Lake Mead. This flow, plus allowable cumulative depletion of Lake Mead throughout the drought period, would constitute water available for lower-basin use on the main river plus one-half of any water granted to Mexico. A study has been made of the depletion of Lake Mead governed by the presumption that all requirements of the lower river will be met by a discharge of 7.5 million acre-ft per yr of water from Hoover Dam. The result is a maximum cumulative depletion of 20.0 million acre-ft in 1936 (see Column (16), Table 2).

Records are insufficient to show how frequently drought periods such as the one of the ten years, 1929–1939, will be repeated. Indications are that four droughts of considerable severity have been experienced in the past hundred years.

Construction of projects in the lower basin, provided for by the Boulder Canyon Project Act, has progressed rapidly and major items are (in 1939) substantially completed. Other, smaller, related projects have been authorized for the lower basin. The following has particular importance: Projects or divisions of projects in the lower basin which will draw their water supply from the main Colorado River are those (a) which are now (1939) fully planned; (b) for which money has been provided at least in part; and (c) upon which construction has begun or is about to begin. When 90% developed, they will require (together with projects at present (1939) operating) an annual quantity of water fully equal to the average available during the ten years, 1929–1939, to the lower basin under the terms of the Colorado River Compact

from the flow of the river. The eventual needs of these projects are estimated herein at 6 200 000 acre-ft per yr, requiring the delivery at Hoover Dam of 7 000 000 acre-ft per yr with the average direct stream flow available at the outlet of the dam of 6 700 000 acre-ft per yr. If this demand is increased and particularly if it is burdened further with an additional grant of water for Mexican lands, Lake Mead will be seriously depleted in a series of drought years such as those through which the United States has just passed and under the conditions of river development which have been assumed.

However, a number of years will elapse before the lower-basin projects will need their full quantity of water. This time has been advanced due to conditions growing out of the years of depression through which the nation has been passing and by failure to realize earlier anticipations. Lake Mead in June, 1939, rose for the first time to the spillway level.

Appraisal of Future Trends

The consequences upon future use of water in the upper basin of the Compact, Boulder Dam Act, deficient stream flow, and accomplished and planned construction on the river may be briefly summarized as follows: (1) Early estimates of surplus, unallotted water are shown to be in error; (2) any increase in the use of water above that planned, and for which construction of works has begun in the lower basin, will require the appropriation of water which it has been presumed has been reserved for upper-basin use; (3) the small, future, financial aid that upper-basin development expected to receive from payments made to the Boulder Dam Fund by users of power and water is of doubtful maturity; (4) large storage volume upon the upper tributaries will be desirable for river regulation before final development is attained; (5) for a number of years large quantities of surplus water will flow into or through Mexico and will be available in part for use there with the possibility (according to the views of some) that use there will follow and finally that rights will be granted by treaty, with resulting ultimate detriment to the upper basin and perhaps to the lower basin. In pursuance of this condition an amicable agreement with Mexico should be made, if possible, protecting the water for United States use; or, failing this, a system for regulating the river should be adopted which will make further irrigation development in Mexico impossible through the resulting river regimen. The last may require the construction of additional reservoirs or, as a temporary expedient, the curtailment of some power development at Hoover Dam; but it is believed entirely feasible.

About 1 500 000 acres of land are now (1939) irrigated in the upper basin of the Colorado River. It is believed that this land consumes about 1.5 acre-ft per acre of water in years of plentiful water supply, or a total of 2 250 000 acre-ft. About 130 000 additional acre-ft have been diverted each normal year from the basin. Estimates of the additional area which, eventually, can be irrigated have been made and total about 2 500 000 acres; but these estimates are open to question, and studies are in progress, principally by the U. S. Bureau of Reclamation, to define, more accurately, the arable, and perhaps potentially irrigable, acreage. The results of recent surveys for the area

drained by the Upper Colorado River in the State of Colorado (excluding the tributaries of the Gunnison and Dolores Rivers) have been made public and may be compared with an earlier estimate of potentially irrigable area in this basin of 290 000 acres. The recent survey gives the arable land in this same basin as 122 830 acres, less than half of the 290 000 acres (see comment on Table 3, in the Appendix). It is said, however, that the 122 830-acre estimate intentionally omits some land irrigable for meadow. Perhaps a similar reduction will be indicated for other subdivisions of the upper basin; but even these recent, more careful, surveys do not vouch for the general economy and feasibility of irrigating all the arable area which they disclose, nor for the adequacy of the water supply available to each area. Therefore, it remains uncertain how much of the 2 500 000 acres of upper-basin land, reported earlier as suitable for irrigation, will be irrigated.

By definition, the Colorado River Compact makes any area supplied with water from the river or its tributaries a part of the river basin even if it may lie without the water-shed (although this tacitly applies only to the signatory States and does not authorize nor encourage diversion of Colorado River basin water outside of States of the basin). The makers of the Compact fully intended that water of the river system should be used either within or without the basin as desirability later dictated. However, diversion of water from the upper basin for use on adjoining areas is now assuming an importance greater than was contemplated. The Colorado-Big Thompson Project is an object lesson to those whose engineering vision is restricted. A few years ago, this diversion (which will cost $44 000 000) was seriously considered only by a few of the most comprehending engineers. Others deemed the possibility of the undertaking as remote. The soundness of great diversions from the basin is due to the presence of highly developed agricultural, urban, and industrial areas just outside the basin which, although separated by mountain rims from the basin, are lower in elevation than much of the basin area. Hence, they have longer growing seasons and also possess abundant, highly productive soils, opportunities for the consumption of large quantities of electric power, and in general offer situations in which water supply has the best opportunity to stimulate an advanced economic and social development.

On the east side of the upper basin, in Colorado, twelve trans-mountain diversions have been operated for a number of years. The most important of these is the Grand River Ditch crossing the continental divide west of Ft. Collins, Colo. Only one is a tunnel, the Busk-Ivanhoe, west of Leadville, Colo., which makes opportune use of an abandoned railroad tunnel, shared with a highway. These diversions are said to have a normal aggregate capacity of 46 000 acre-ft per yr. The Moffat Water Tunnel Diversion was placed in operation in 1937. This structure is for the benefit of the City of Denver, Colo., and can divert 80 000 acre-ft per yr. The Jones Pass tunnel diversion, at the head of Clear Creek west of Idaho Springs, Colo., for the City of Denver has a capacity of 20 000 acre-ft per yr. This water is used to boost the low-water discharge of the South Platte River in the vicinity of Denver to avoid excessive pollution of this stream by the effluent from the Denver Sewage Treatment Works. After this use, it serves for irrigation and, when not re-

quired to increase the discharge of the river, may be turned into the Denver municipal supply. A notable tunnel trans-mountain diversion, constructed in 1938, is the Independence Pass (or Twin Lakes) which brings 48 000 acre-ft per yr into the head-waters of Arkansas River southwest of Leadville. The most pretentious diversion project that has reached the construction stage is the aforementioned Big Thompson project. It has many features and will divert 300 000 acre-ft of water beneath Rocky Mountain National Park for wide-spread benefit of the irrigated area of north-central Colorado. Power obtained from the falling water on the east side of the mountains is an important factor in the usefulness of the project.

Besides the trans-mountain diversions which have been noted, more than 17 others situated throughout the length of the crest of the Rocky Mountains in Colorado and dipping into New Mexico have been studied. Several studies have included field surveys and hydrography. Some of the projects are large.

Along the west rim of the upper basin, in Utah, nine diversions across the water-shed have been operated for several years. Much more important than the others is the Strawberry Tunnel which transports about 50 000 to 70 000 acre-ft per yr into the Utah Valley. In 1938, two, 1-mile diversion tunnels were built east of Spring City and Mount Pleasant, Utah, to transport 4 000 acre-ft each into the San Pitch Valley. These supersede two of the small earlier diversions. The Deer Creek project includes, as one of its features, a 5.5-mile tunnel to divert 32 000 acre-ft from the Duchesne River to the head of the Provo River, 35 miles east of Park City, Utah. More than eight other projects for diverting water from the upper basin to valleys lying to the west have been studied. In one of the most important of these it is planned to divert water from the Green River north of the Uinta Mountains to the Bear River, Wyoming. Investigations of two alternate plans including field surveys have been made by Mr. Ralf R. Woolley of the U. S. Geological Survey. A project of even greater magnitude has been studied by Mr. Kenneth Borg. This project would transport water from the Green River south of the Uinta Mountains and deliver it for use in the Salt Lake and adjacent valleys.

It is likely that development within the water-shed of the upper basin will progress in the form of a number of projects of moderate or small size. A considerable number of these have been given study. Localities exist within the upper basin where special crops and fruits grow luxuriantly. The produce is of such high quality that it is possible to overcome the handicap of remoteness and command a premium for it on national markets. Localities of this nature deserve expansion. Other localities, particularly the stream valleys at high elevations situated close to mountain and high plateau livestock ranges, support a productive livestock industry. It is said about half the feed for livestock in these areas comes from the ranges and half from cultivated fields. Western livestock raising is a highly specialized industry conducted mainly by men of great skill who have spent their lives in this occupation. When properly organized and managed, individual units may be very profitable; and extension of irrigation for the benefit of this industry, even at a cost that seems somewhat out of proportion to the low current prices for hay crops, is justified. Since 1936 the open range has been under close regulation; its use has been curtailed

and made permissive in order to preserve its feeding capacity. Furthermore, use of the range has been made conditional upon the possession of commensurable property from which feed may be produced to supplement, to an extent more than required by natural conditions heretofore, the feed produced by the range. In this manner additional irrigated land for the production of livestock feed is assuming a new importance.

Developed water power in the upper basin of the Colorado River totals about 50 000 installed horsepower; and nearly all of it is along the Upper Colorado River in the State of Colorado where power-plants are within transmission distance of the Denver area. It is estimated that the upper basin can produce 2 000 000 primary horsepower with a regulated stream flow. Power sites on the Green River and main channel of the Colorado River, in their courses between Green River, Wyoming, and Lee's Ferry, are capable of producing the bulk of this output. Several gigantic dams would be required and the power would necessarily be made available in large blocks, with large initial investments being required. The power that can be developed will be lessened by future up-stream depletion due to irrigation development and the diversion of water from the basin. Proposals to place long reaches of the river and adjoining lands in national parks constitute a threat to the development for power and perhaps other purposes of the Green and Colorado rivers, in Utah. The necessity, or desirability, of making park reservations of these lands may be seriously questioned. Although they constitute scenic features of great importance and magnitude, and although money should be spent to provide means of access and travel, the natural scenery is not of a nature that it can be marred appreciably by such unrestricted human habitation and use as it is likely to undergo. Rather the few ranches, prospectors and nomad Indians who inhabit the country, add much to its picturesqueness. Dams and conduits that may be built will be only tiny spots in a vast wilderness of canyons and plateaus, largely of bare sandstone and shale. The few individual features requiring protection can be individually reserved without laying the "heavy hand" of park reservation over the entire area.

Conclusions

Conditions which will justify the development of the large power sites are unforeseen at present. Two possibilities are the construction of major dams for river control with the generation of power as a by-product, for sale at a price so low as to attract, to the vicinity, industries in which the cost of power bulks large; or an unforeseen technological development favorable to the development of power in this region.

This paper shows the development of the Colorado River in the upper basin to be varied and complex. Units in this development may be approximately grouped into: (1) Normal irrigation enterprises for assorted farming products; (2) irrigation of lands to produce livestock feed and meadow; and (3) the diversion projects. This classification is mainly an economic one.

Many of the undertakings for using water in recent years (and, most likely, many of those to be undertaken in the future) will furnish additional water to

projects already operating. For some operating projects stored water is needed for late-season use when stream flow is normally low. Other operating projects exhibit the habits, not uncommonly displayed by western irrigation projects, of having a water supply that was initially over-estimated and hence unreliable, or having a canal system extended to serve lands for which no adequate supply of water is available; and under these conditions a supplementary supply of water is needed. In general, projects for supplying supplementary water where this is needed are regarded as particularly desirable because a small increment of investment commonly produces a relatively large increment of utility or return.

The proposed large diversions from the basin are especially complex and are the objects of particular engineering interest. Each includes a number of features such as: Collecting works; dams for diversion; storage reservoirs (for (*a*) water diverted, (*b*) water to supply deficiencies in lower reaches of the stream from which the diversion is made, and (*c*) to redistribute water near the points of use); pumping plants; electric generating stations; transmission lines; long tunnels; and, the more usual conduits and water control structures. The variety of purposes served by these projects includes irrigation supply, water for industrial and varied municipal purposes, improvement of recreational features, and the generation of power. With some projects should be included the functions of drainage, protection of land from torrential floods, and recovery of seepage, waste and return flow from low streams, drains or lakes. Some may serve the supplemental purpose of transporting power in the form of flowing water at a high elevation to a point where the generation of electricity can occur closer to markets and where greater power, by virtue of higher head, can be recovered.

It is an accepted fact that power generation is to play an important rôle in water development in the West. This is true in the upper basin of the Colorado River both for the projects within the basin and for the large diversions from the basin. Power generating stations, transmission lines, and pumps, free the energy gradient of the combined stream from many topographic limitations which would have to be met if water conduits alone were used. Increased efficiencies of electrical and hydraulic equipment now available and their lower cost and greater reliability—all coupled with the fact that a power system created as a part of water development and built primarily to serve this purpose, often does not need to meet separate development costs or the service requirements of systems built for other purposes—increase the usefulness of such power adjuncts to water projects. Then too some water development projects provide surplus power to sell at a profit to pay part of the cost of water development construction. However, if this is to continue to be an effective method of financing water projects, those interested in them must protect the power market and discourage abortive attempts to obtain cheap power through outside subsidy which, in turn, reduce or destroy the possibility of power development subsidizing water development.

The conviction is held by many that the water of the Colorado River System in the upper basin should be used in ways and places that will, most effectively, build communities with a broad industrial base, including not only

agriculture but the winning and fabrication of mineral products and use of other industrial resources of the region. To the extent that this can be accomplished will come wide-spread advantages not only for residents of the basin and adjoining areas but for all areas within trading distance. This area may be indefinite in extent but it includes, more especially, the Pacific Coast and the Great Plains States. Communities dependent upon agriculture, supplemented by other industrial production, appear to have a more promising future in the inter-mountain west than communities dependent upon agriculture alone. This general thesis is further justification for the use of available water supplies to expand communities already established and experiencing healthy growth as a result of a combination of good agricultural conditions surrounding them, favorable position as trade centers, and possession of resources and facilities for other forms of industrial development. It is also a further justification for the great exportations of water from the basin which are being undertaken and proposed, and for the relatively high cost of these projects and the initial financial subsidization they need.

The general planning of the development of the Colorado River and its tributaries in the upper basin will continue. Such general plans are subject to drastic modification as new conditions develop and new information is available. Some past results are readily accessible in *Water Supply Papers* of the U. S. Geological Survey and various reports prepared by the U. S. Bureau of Reclamation and agencies of the States concerned. There remains, as distinct from general planning, a vast mass of engineering planning, of a detailed nature: (*a*) to unravel the hydrographic and other engineering complexities of individual projects; (*b*) to devise better and more economical combinations of their features; and (*c*) in short, by engineering ingenuity, to bring individual projects within the limits of an assured feasibility. The U. S. Bureau of Reclamation is undertaking a part of this heavy burden, successfully, and representatives of States and areas which are to participate in the benefits should assume another part of it both to bring more resources to the study and to insure realization of the opportunities which are within their grasp.

Acknowledgment

Fig. 1 has been reproduced from Drainage Basin Studies of the Water Resources Branch of the National Resources Committee.

APPENDIX

Supporting Data

Future Water Supply

Table 1 contains an estimate of the future water supply to be drawn from the main channel of the Colorado River, below Lee's Ferry. The following notes refer to corresponding items and column numbers in that table:

Item 8, Column (3).—Includes losses of the main canal through Mexico.

Item 10, Column (3).—This quantity is water measured to irrigators in Mexico and does not include the transmission losses that will be incurred through the continuation of the Mexican Canal in operation after water for the Imperial Valley is drawn through the All-American Canal.

Items 3 and 4, Column (4).—The smaller acreage noted lies within the valley and is likely to be developed by the present canal system. The larger acreage includes good land on the mesa to the west of the Palo Verde Valley, which may be irrigated with a moderate pump lift.

Items 5 and 6, Column (4).—The smaller acreage is for the first unit of the Gila Project lying east of Yuma. It is reasonable to suppose that, when completed, this unit will be developed to a moderate degree within a few years. The entire Gila Project includes about 500 000 acres maximum; but other proposals have been made for the irrigation of the lands of the Gila River Basin of Arizona from the Colorado River and the large estimate given by Item 6 is perhaps the greatest aggregate acreage of these proposals.

Item 7, Column (3).—This is the quantity of water diverted for irrigation and excludes water diverted for power and other uses.

TABLE 1.—Approximate Estimate of Future Water Supply to be Drawn from the Main Colorado River, Down Stream from Lee's Ferry

Item	Development (1)	Acres irrigated since 1937 (2)	Water diverted in recent years, in acre-feet (3)	Future irrigated area, in acres (4)	Future demand for water, in acre-feet per year (5)
1	Colorado Indian Reservation	7 500	....	100 000	270 000
2	Colorado River, Los Angeles Aqueduct	....	....	....	1 000 000
	Palo Verde Irrigation District:				
3	Developed by present canal system	23 000	225 000	75 000	....
4	By total anticipated development	....	....	115 000	200 000
	Gila Project:				
5	First unit, east of Yuma, Ariz.	....	....	150 000	....
6	Completed project	....	....	2 900 000	405 000
7	Yuma Project	60 000	450 000	80 000	325 000
8	Imperial and Coachella valleys	450 000	3 000 000	1 000 000	4 000 000
9	Total	....	....	....	6 200 000
10	Mexico	....	850 000	....	....
11	Hoover Dam losses	....	....	....	500 000
12	Net river losses below Hoover Dam	....	....	....	600 000
13	Sluicing and incidental loss at, and below, Imperial Dam	....	....	....	200 000
14	Power and irrigation losses, Lee's Ferry to Hoover Dam	....	....	....	300 000
	Gain, Lee's Ferry to Hoover Dam:				
15	Minimum	....	....	....	200 000
16	Maximum	....	....	....	700 000

Heading.—In the preparation of values of estimated future demand for water, attention has been given mainly, in the case of irrigation projects, to the indications afforded by physical features of the area served and of the construction completed, under way, or authorized. Thus constructed canal capacities and present experience with irrigation have been given more weight in arriving at values than general agreements or understandings regarding the

distribution of water. But in all cases the figures adopted are intended to be conservative in not overstating the quantities.

Those familiar with Colorado River agreements may note some discrepancies between stated future demands for water and certain stipulations that have been made.

Discussion of Column (5), Table 1

Item 1, Column 5.—This estimate is based on 90 000 acres at 3.0 acre-ft per acre. Return flow from the Indian Reservation will re-enter the Colorado River for use below. A contract has recently been let for the diversion dam to serve this project.

Item 2.—The capacity of the Colorado River aqueduct is to be 1 640 cu ft per sec or 1 180 000 acre-ft per yr but it is intended to operate at 1 500 cu ft per sec or 1 090 000 acre-ft per yr. The date for the attainment of full operation was set by earlier forecasts at about 1970.

Item 4.—This estimate is based upon 90% of 75 000 acres at 3.0 acre-ft per acre. Return flow from the Palo Verde Irrigation District will re-enter the Colorado River for use lower.

Item 6.—This estimate is based upon 90% of 100 000 acres at 4.5 acre-ft per acre. It is to be noted that this acreage is less than two-thirds the acreage announced for the first unit of the Gila Project. Also return flow from this project will not be available, without pumping, to any United States irrigation undertaking.

Item 7.—This estimate is based upon 90% of 80 000 acres at 4.5 acre-ft per acre. Return flow from this project will be available only for use in Mexico. It is to be noted that the amount of water estimated for this project is less than the amount now being used on the project as indicated by the diversions made to the project less the large quantity of water returned more or less directly to the river after its use for power and maintaining optimum velocities and depths in a portion of the canal system. The present indicated use of water on this project is considerably greater per irrigated acre than the basis of estimates appearing in the foregoing table and this is true of the use of water on other irrigation projects diverting from the lower Colorado River.

Item 8.—This estimate is based upon 90% of 1 000 000 acres at 4.5 acre-ft per acre. The canal considered is the All-American Canal which will supply water to the present Imperial Valley, extensions to this within the Imperial Irrigation District, to the East and West Mesas adjoining, the Coachella Valley, and possibly to San Diego, Calif., and vicinity, and other areas. The capacity of this canal (beyond Pilot Knob) is 10 000 cu ft per sec or 7 300 000 acre-ft per yr; 4 000 000 acre-ft are 55% of the canal capacity. Considering canal capacity to be 11% greater than the discharge (average) of the maximum month, the ratio of water diverted during the year to the annual canal discharging capacity is nearly 0.60 for the Imperial Canal and Yuma Main Canal (irrigation use only) within recent years.

Item 11.—This item is mainly evaporation. The value stated has been widely used as the evaporation from the reservoir some decades hence when the reservoir has been considerably silted and perhaps somewhat contracted in area. No doubt, the present evaporation is larger than this amount and

there is some possibility that it is larger than is generally believed and will be larger in the future than the amount stated (for more information see Column (10), Table 2).

Item 12.—This is mainly evaporation from the river surface, adjacent flood plain, and Parker Reservoir to which is added stream inflow between the points considered. The latter is not great and the result is a rather large net loss as indicated. This net loss has been estimated as much as 200 000 acre-ft per yr greater than is given herein and may be considerably higher when other reservoirs are constructed on the lower river.[3]

Item 13.—It is expected that some loss of water from the Colorado River to the Gulf of California cannot be avoided. It is also considered by some that sluicing down the river of silt removed from All-American Canal water at Imperial Dam will require considerable water. Official estimates of water for incidental losses and sluicing are about twice the value stated in Table 1.

Item 14.—This amount includes allowance for evaporation from reservoirs which in the future are expected to be constructed on the Colorado River between the Hoover Dam and Lee's Ferry and for irrigation development on tributaries entering the river between these same two points. The two most important tributaries are the Little Colorado and the Virgin rivers. Official estimates are more than twice the amount given.

Item 15.—The gain from Lee's Ferry to Hoover Dam Outlet includes the discharge of the Little Colorado River, the Virgin River, and numerous smaller streams and springs. Evaporation from the river must be deducted and this may be from 50 000 to 100 000 acre-ft per yr. The net annual average gain has been variously estimated between the two values given. For reasons explained under Column (10), Table 2, the writer's estimate is near the low value.

Distribution of Water Between Upper and Lower Basins

Table 2 should not be interpreted as indicating how the water of the Colorado River will be distributed when development has reached a maximum, and during a dry period. Rather it is intended to show what restrictions the Colorado River Compact places upon the distribution of the water under these conditions. A modification in compact provisions may eventually appear desirable.

The table was prepared for water years 1922 to 1937 which include the very dry period from 1931 to 1934 with sufficient years before and after to be indicative. There seemed to be no harm in setting forth presumptive data for later years to indicate what results would follow and the table has been extended to include these and derivations from them. It must be kept in mind, however, that all data for 1938 and following years are imaginary. Also it should be noted that the discharges assumed for 1938 and following years are high compared with the averages for periods passed. It is almost trite to state that if these discharges are as high as estimated, the conditions contemplated by the Colorado River Compact would be more quickly restored and less alarm would be warranted over shortages which would occur during such a dry period as centered about 1931 to 1934. If anything, Table 2 is slightly biased in favor

[3] The basic data for this estimate are from the Debler (U. S. Bureau of Reclamation) Report of 1934.

of a larger usable water supply than will be realized from the flow of the river under conditions of the years considered. Explanations of the individual columns follow.

TABLE 2.—DISCHARGE OF THE COLORADO RIVER AT LEE'S FERRY, PRESENTING DISTRIBUTION OF WATER BETWEEN UPPER AND LOWER BASINS AS REQUIRED BY THE COLORADO RIVER COMPACT

(All Values in Millions of Acre-Feet per Year)

Year	Measured discharge, in millions of acre-feet per year	Ten-year cumulative discharges	Estimated Annual Depletion		Columns (2) and (4), minus Column (5)	Ten-year cumulative values from Column (6)	Measured discharge at Bright Angel, Ariz.	Ten-year cumulative values from Column (8)	Column (8) minus Column (2)	Annual estimated deficiencies	Water available for consumptive use	Annual discharge at Lee's Ferry based on conditions affecting Column (12)	Annual water supply, Lower Colorado River	Departures from cumulative sums of Column (14)	Cumulative depletion of Lake Mead
			Actual	Theoretical Future											
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1922	16.4	....	2.4	7.9	10.9	....	....	....	....	....	....	10.9	....	....	....
1923	16.1	....	2.5	8.1	10.5	....	17.0	....	0.9	....	....	10.5	10.6	2.5+	....
1924	12.5	....	2.1	6.8	7.8	....	13.0	....	0.5	....	6.8	7.8	7.5	1.9+	....
1925	11.3	....	2.2	7.1	6.4	....	11.8	....	0.5	....	7.1	6.4	6.1	0.1−	....
1926	14.0	....	2.2	7.2	9.0	....	14.4	....	0.4	....	7.2	9.0	8.6	0.4+	....
1927	16.5	....	2.5	8.3	10.7	....	17.3	....	0.7	....	8.3	10.7	10.6	2.9+	....
1928	15.3	....	2.3	7.5	10.1	....	15.6	....	0.3	....	7.5	10.1	9.6	4.4+	....
1929	19.2	....	2.7	8.8	13.1	....	19.4	....	0.2	....	8.8	13.1	12.5	8.8+	....
1930	13.1	....	2.2	7.1	8.2	....	13.4	....	0.3	....	7.1	8.2	7.7	8.4+	3.8
1931	6.4	140.8	1.7	5.6	2.5	89.2	6.7	....	0.3	....	5.6	2.5	2.0	2.3+	9.3
1932	15.3	139.7	2.4	7.7	10.0	88.3	16.0	144.6	0.7	....	7.7	10.0	9.9	4.0+	6.9
1933	9.7	133.3	1.9	6.4	5.2	83.0	10.0	137.6	0.3	....	6.4	5.2	4.7	0.6+	9.7
1934	4.4	125.2	1.5	5.0	0.9	76.1	4.7	129.3	0.3	....	5.0	0.9	0.4	7.1−	16.8
1935	10.0	123.9	2.0	6.5	5.5	75.2	10.2	127.7	0.2	....	6.5	5.5	4.9	10.3−	19.4
1936	11.9	121.8	2.2	7.1	7.0	73.2	12.3	125.6	0.4	−1.8	7.1	7.0	6.6	11.8−	20.3
1937	11.9	117.2	2.2	7.1	7.0	69.5	12.4	120.7	0.5	−3.7	5.3	8.8	8.5	11.4−	19.3
1938*	14.0	115.9	2.2	7.2	9.0	68.4	14.5	119.6	0.5	−1.1	3.5	12.7	12.4	7.1−	....
1939*	15.0	111.7	2.3	7.5	9.8	65.1	15.5	115.7	0.5	−3.3	6.4	10.9	10.6	4.6−	....
1940*	15.0	....	..	..	10.0	66.9	15.5	....	....	....	4.2	13.3	13.0	0.3+	....
1941*	....	....	..	..	10.0	74.4	....	....	....	....	....	....	....	....	....
1942*	....	....	..	..	10.0	74.4	....	....	....	....	....	....	....	....	....

* All values below line (for 1938 and following years) imaginary.

Column (1).—The water year beginning October 1 of the previous year and ending September 30 of the year given. All data are for water years.

Column (2).—Measured discharge of the Colorado River at Lee's Ferry, in millions of acre-feet for the water year.

Column (3).—Ten-year cumulative discharges of the Colorado River at Lee's Ferry. The discharge for the year in question is added with the discharge for the nine preceding years.

Column (4).—Estimated annual depletion of the Colorado River above Lee's Ferry due to all present uses and diversions. To and including the year 1934, the depletions given were obtained from the Debler Report[3] on the water resources of the Lower Colorado River. This is the most complete and authoritative source of information. The writer has extended these estimates for 1935, 1936, and 1937 using the formula employed by the Bureau of Reclama-

tion. The Debler Report gives values for calendar years but it is believed they are applicable without appreciable error to water years.

Column (5).—Estimated annual depletion in the upper basin of the Colorado River if development had proceeded to the limit imposed by an average use of 7 500 000 acre-ft per yr. Values for 1934 and earlier were obtained from the Debler Report.[3] Those for 1935, 1936, and 1937 were estimated by the writer following the Debler method. It is to be noted that values in this column vary from year to year with the dryness of the season.

Column (6).—Annual discharge remaining at Lee's Ferry after subtracting the values of Column (5) from the sums of the values of Columns (2) and (4). These latter sums are the total yield of the Colorado River and tributaries above Lee's Ferry. The items of Column (6) are amounts which would be available at Lee's Ferry to the lower basin had "full" upper-basin development existed during the time covered by the table, and without consideration for the provision of the Compact which required 75 000 000 acre-ft to flow past Lee's Ferry in any 10-yr period.

Column (7).—Ten-year cumulative sums of values of Column (6). It is these values that the Compact requires shall not be less than 75 000 000 acre-ft. Note that the values are less than the required 75 000 000 acre-ft for 1936 and 1937 and are likely to be less in succeeding years until 1943. The information in this column indicates that development in the upper basin cannot be based reliably upon an average consumption of 7 500 000 acre-ft of water per yr throughout a series of dry years but must be based upon some less amount; and that there is no surplus water available from the Colorado River over and above the allotments made by the Compact to the upper and lower basins.

Column (8).—Measured discharge of the Colorado River at Bright Angel, Ariz. Bright Angel is midway between Lee's Ferry and the Hoover Dam and below the mouth of the Little Colorado River, which supplies the principal inflow between the two points specified.

Column (9).—These are 10-yr cumulative values of the discharge at Bright Angel.

Column (10).—Differences between measured annual discharges at Lee's Ferry and Bright Angel. The net gain in the Colorado River between Lee's Ferry and Hoover Dam outlet before deducting evaporation from Lake Mead has been taken numerically equal to these differences in later computations.

The smallness of the gain according to this assumption is somewhat surprising particularly as there is a large drainage area between the points considered. The Virgin River discharges from 100 000 to 400 000 acre-ft per yr into Lake Mead, and measurements by E. C. LaRue, M. Am. Soc. C. E., during August and September, 1923, disclose streams discharging an aggregate of 272.3 cu ft per sec into the Colorado River between Bright Angel and Black Canyon. Furthermore, a comparison of river discharge measurements made at Black Canyon and Bright Angel in 1932, 1933, and 1934 shows net river gains between these points of 200 000 to 400 000 acre-ft per yr. These years immediately precede the impounding of water in Lake Mead.

However, comparisons of the discharge of the Colorado River at Bright Angel and the Virgin River at Littlefield, Ariz., with the discharge from Hoover

Dam, and accumulated visible storage in Lake Mead since the impounding of water began and during the period while the reservoir has been filling, show a remaining apparent loss of water after making a generous allowance for bank storage and a customary allowance for evaporation associated with Lake Mead, for most months and years. This may be the result of inaccuracies in discharge measurements, errors in the stated volume-stage relation for Lake Mead, too low an estimate of evaporation, temporary or permanent seepage from Lake Mead greater than was expected, or any combination of these factors.

A close study seems to indicate that: There is a very satisfactory consistency in the data which shows that discharge measurements are to be relied upon as being sufficiently accurate; there may be bank storage or seepage greater than estimated (10 ft of depth for the water area); there is a likelihood that the volume and area of the reservoir are greater for a given stage than has been stated heretofore; there is also a likelihood that evaporation is greater than estimated (6 ft of depth per yr) both because unit evaporation is greater and the area of the reservoir is greater; and, the apparent excess loss has ranged from 200 000 acre-ft in 1936 to 800 000 acre-ft in 1938. Inasmuch as part of this apparent loss is likely to prove to be an actual loss (evaporation and seepage other than bank storage)—perhaps of the order of the discharge of the Virgin River into Lake Mead (which is measured and was credited in the computations)—the net gain between Bright Angel and the outlet of Hoover Dam will continue to appear to be zero if the evaporation from Lake Mead is considered the same as before (at normal stages 500 000 acre-ft per yr when the reservoir area is somewhat decreased by silting).

This is an awkward manner of estimating the water resources of the river but appears to give correct results; and, until the volume-stage-area data for Lake Mead are confirmed or corrected, and until more data concerning the hydrology of the reservoir become available, no change in method is justified.

Column (11).—Annual quantities by which discharge passing Lee's Ferry would fail to provide a 10-yr cumulative discharge, at this station, of 75 000 000 acre-ft for any 10-yr period and under the conditions contemplated in the preparation of data for Columns (5), (6), and (7)—that is, an upper-basin development based on an average annual use of 7 500 000 acre-ft of water. The values of Column (11) are obtained by noting the quantities in Column (7); and when these become less than 75 million, taking (in chronological order) differences between adjacent values. It would thus become apparent at the end of water year 1936 that the discharge of that year had been so low that the cumulative discharge for it and the nine preceding years had dropped to 73.2 million acre-ft; but, had the discharge at Lee's Ferry been 1.8 million acre-ft higher than it was assumed to be in Column (7), the cumulative discharge for 1936 and the nine preceding years would have been exactly 75 000 000 acre-ft. The 1.8 is the difference between 73.2 and 75.0.

Column (12).—Water available annually in the upper basin for consumptive use, based upon a development using 7 500 000 acre-ft per average yr except when this must be diminished to sustain the 10-yr cumulative discharge at Lee's Ferry at the 75 000 000 acre-ft specified by the Colorado River Compact. No shortage in the 75 000 000 acre-ft cumulative amount occurred until 1936;

but this could not have been known until the end of 1936 and, therefore, the restitution of the shortage made in 1937. Similarly, it is considered that the restitution of the shortage of 1937 would be made in 1938, and so on. The values of Column (12) are obtained by subtracting from the values of Column (5) the values of Column (11) which appear on the lines next above. It is to be noted that the 10-yr-cumulative-amount provision of the Compact, if strictly adhered to, causes the greatest shortages to occur in the upper basin a matter of years after the end of the dry period. The only substantial effect of following this provision would be to hasten the restoration of the level of Lake Mead or other reservoir. The average of values of the column for the years 1931 to 1940, inclusive, is 5.8 million acre-ft.

Column (13).—Annual discharge at Lee's Ferry under the conditions upon which the data in Column (12) are based. Values are obtained by adding to the values of Column (6), the values of Column (11) appearing in the line next above (the sign is not considered). It is to be noted that under the conditions governing, very little water would be contributed to Lake Mead (or other reservoir on the lower river) in 1931 and 1934.

Column (14).—Annual water supply to the lower Colorado River reckoned at the outlet of Hoover Dam. Conditions of Columns (12) and (13) govern. To obtain the values of this column, the river gains of Column (10) are added to the discharges at Lee's Ferry given in Column (12); 0.8 million acre-ft are subtracted therefrom, representing a future depletion of 0.3 million acre-ft per yr between Lee's Ferry and Hoover Dam, and 0.5 million acre-ft, representing the annual loss from Lake Mead. The average of values of the column for 1928 to 1937, inclusive, is 6.7 million acre-ft. For further information on the probable evaporation from Lake Mead and considerations regarding the gain between Lee's Ferry and the outlet of Hoover Dam refer to the explanation of Column (10).

Column (15).—Departures of the cumulative sum of the values of Column (14) from the cumulation of the average for all years, applied successively each year. This is an indication of the storage requirements for equalizing flows of Column (14) for the entire period.

Column (16).—Likely cumulative depletion of Lake Mead during a drought period similar to the one of 1930 to 1937 after the Colorado River is developed in accordance with conditions stated for the data of Columns (12), (13), and (14). In the preparation of Column (16) the assumption is made that the requirements of the lower basin will necessitate the release of 7.5 million acre-ft per yr from Hoover Dam lake. This will include half of a possible concession to Mexican lands, the other half of which would move through the reservoir, and would be an additional release; but it is not considered in the present calculations because it would be from upper-basin allotments.

The available water from direct stream flow for lower-basin use as given in Column (14) is sufficient to supply the demand of 7.5 million acre-ft per yr without considerable depletion of Lake Mead until the end of the water year 1929; and the reservoir may be considered full at the termination of the spring flood of that year. A start therefore is made in 1930. This year is credited with flow available for the year and from this is deducted 4.0 million acre-ft

representing the depletion of the reservoir for power and irrigation during the late summer and fall of 1929 and the spill required to obtain flood-storage volume before the expected spring flood of 1930; then 7.5 million acre-ft, representing the draft for 1930, is deducted. The item in Column (16) then is depletion of the reservoir at the end of the water year. For each subsequent year a credit is made for the flow available, and a deduction of 7.5 million acre-ft is made. This assumes that no spill, waste, or exclusively power use of water will be permitted under drought conditions.

The sum of the following quantities (each in million acre-ft) is 7.5 million acre-ft: Consumptive use by projects in the lower basin whose construction has begun, 6.2; channel losses between Hoover Dam and Yuma, 0.6; waste at Imperial Canal not recoverable in Mexico, 0.2; and, concession to Mexico, 0.5.

Such a drought coming several decades from now, when full development can be expected, would find Lake Mead partly silted. Perhaps its present capacity of 30 million acre-ft would be reduced to 25 million acre-ft. A depletion of 20 million acre-ft would leave 5 million acre-ft in the reservoir, with a head of 293 ft available at the power plant. This is a little more than half the normal head.

Distribution of Irrigated, Irrigable and Arable Land

In Table 3, the data in Columns (2), (3) and (4) were presented by Daniel W. Mead, Past-President and Hon. M. Am. Soc. C. E., in 1929.[4] They are

TABLE 3.—Estimates of Irrigated, Irrigable, or Arable Lands in the Basin of the Colorado River above Lee's Ferry, in Acres

Item	Basin of the:	D. W. Mead			F. E. Weymouth		
		Irrigated in 1920	Additional irrigable or arable	Total	Irrigated in 1922	Additional irrigable or arable	Total
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	Green River	643 000	1 212 000	1 855 000	532 000	1 067 000	1 599 000
2	Upper Colorado River	542 000	412 000	954 000	588 000	890 000	1 478 000
3	San Juan River	157 000	729 000	886 000	156 000	653 000	809 000
4	Remaining parts of the upper basin	128 000	102 000	230 000	171 000	130 000	301 000
5	Totals	1 470 000	2 455 000	3 925 000	1 447 000	2 740 000	4 187 000

mainly census data and estimates that were available to the Colorado River Board. The data in Columns (5), (6), and (7) were compiled by the engineering staff of the U. S. Bureau of Reclamation under the direction of Frank E. Weymouth, Hon. M. Am. Soc. C. E. They are from a variety of sources including the U. S. Census Bureau.

The two sets of data are not strictly comparable for individual basins because divisions of the basins adopted are not the same; but they should be comparable for the entire upper basin (see Item 5, Table 3).

[4] "The Colorado River and Its Development," by D. W. Mead, *Journal*, Western Soc. of Engrs., July, 1929.

Data for arable lands in the basin of the Upper Colorado River have been cited in a report of the Bureau of Reclamation on the Colorado-Big Thompson Project. This report and the surveys of arable land within this basin were made under the immediate supervision of Porter J. Preston, M. Am. Soc. C. E. For the basin of the Upper Colorado River in Colorado, excluding the Gunnison and Dolores river basins, comparisons with the Weymouth Report are as follows:

Report	Area, in acres
Porter J. Preston:	
Irrigated about 1935	256 300
Additional irrigable or arable	122 830
Total	379 130
Frank E. Weymouth:	
Irrigated in 1922	245 000
Additional irrigable or arable	290 000
Total	535 000

In the Preston Report the land is classified as arable but not necessarily irrigable from the standpoint of economy or adequacy of local water supply. Some areas which are said to be irrigable for meadow feed for livestock were omitted from the estimate intentionally. The Weymouth data for this basin are directly comparable in regard to the basin limits; but they include irrigated pasture land that was omitted from the Preston Survey.

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PAPERS

FIELD TESTS OF A SHALE FOUNDATION

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Synopsis

Field tests described in this paper were conducted on the shale foundation of Possum Kingdom Dam on the Brazos River near Mineral Wells, Tex. They included time-settlement tests on various sizes of square plates, a cyclic loading test, and a series of tests to determine the effect of bond and coefficient of friction of concrete monoliths on a prepared shale foundation. The tabulated and graphical field data presented are analyzed in an attempt to verify some of the current theories advanced by students of soil mechanics. Comparison is also made between somewhat similar tests performed in the laboratory on smaller specimens. Formulas and permissible load values are suggested for this particular foundation material.

Introduction

The necessity of placing dams on difficult foundations is becoming increasingly frequent as the better sites are put to use and as more and more control and conservation of water is demanded by the public. The engineer is thus faced with the problem of using a "second best" or a "third" or "fourth best" site, as the case may be.

Possum Kingdom Dam, near Mineral Wells, is one of several on the Brazos River designed to control floods and conserve water for useful purposes. Design requirements as set up contemplated a masonry dam for a head of 130 ft, founded in the blue shale underlying the sand and gravel in the river-bed at a depth of approximately 20 ft. The procedure adopted was the usual one of examining available literature on shale foundations, performing specific tests and experiments on selected samples in the laboratory, seeking advice from experts and consultants and then making field tests under natural conditions. In due course certain information was acquired and, more important, certain preconceived notions as to the extent of available information and the consistency of test results acquired with identical technique were proved to be in error. Many of these curious anomalies in the data cannot be explained. This paper is not intended to be a profound dissertation on soils mechanics or

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by January 15, 1940.

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foundation theories; it is confined to a description and record of field tests and such observations and deductions as seem to be warranted by the results.

The literature on shale foundations gives an excellent "bird's-eye view" of the problem and serves as a reassurance that such foundations have been utilized successfully in the past. All too frequently, however, reports of tests are not sufficiently comprehensive enough, are vague in description, or lack conclusions that could have been substantiated by later measurements. For maximum benefit to the profession the important boundary conditions of a problem should be stated, with a description of the material, a full record of the tests, the final working loads used in the design of the structure and, if available, the deformation of the foundation under actual loads. This last requisite cannot be furnished by the writer but it is hoped that it may be partly supplied by others during discussion.

Purpose of Field Tests

The general purpose of the field tests was to obtain data on the probable movement of the masonry dam on the shale foundation when subjected to sustained loads and a repetition of loads. This general purpose was divided into five component parts, as follows: (1) To study the sliding of concrete placed on a prepared shale foundation, under the influence of horizontal and vertical loads; (2) to observe the fatigue effect on the shale when the load varied for several cycles from a maximum to a minimum unit load; (3) to determine the time effect on an area of the shale under the weight of a constant load; (4) to establish the relationship between various sizes of loaded areas under equal unit loads and the settlement of those areas; and (5) to compare values from large-scale models loaded under natural conditions with similar values obtained in the laboratory on selected specimens, under assumed and controlled conditions of drainage and temperature.

At the time the tests were proposed, it was realized that conclusive data on each of the foregoing items might not be obtained and that supplementary points of interest would occur to the investigators during the tests. Both of these things happened and the result was a progressive accumulation of knowledge. Fundamental assumptions made prior to the tests included the observation that failure would probably not occur from compressive stresses in the shale but from the tendency of the dam to slide on the contact plane between the masonry and the rock. This item was given primary consideration. Another fundamental assumption was that the shale was less rigid than the concrete dam above it and that the deformations would be confined to the shale.

Geology

The rocks at the dam site are sedimentary in character, consisting of a top member of limestone and grading downward through various admixtures of sandstone, silty and sandy shale, conglomerate and finally a fine-grained blue shale. The blue shale bed is approximately 80 ft deep and is supported from below by a hard gray sandstone of unknown depth. This general stratification is apparent on the cliff on the left bank and in the trenches that were dug in the sloping ground on the right bank of the river. The Brazos River has cut through these

rock barriers successfully and is now about 300 ft below the top of the limestone cliff. The cemented conglomerate and the sand and gravel in the river-bed have probably been brought down from the headwaters during flood periods and deposited at the site.

The top part of the Wolf Mountain shale member immediately below the base of the overlying limestone and conglomerate has been oxidized and altered by waters that have seeped through the relatively porous limestone and conglomerate and have followed along the top of the impervious shale. The color has been changed by oxidation of the iron content from gray to yellow; calcareous cement has been dissolved and the shaley content softened to clay. Except for this weathered zone at the top of the formation and a moderate general penetration of surface weathering, the sandstone, sandy and silty shales, and blue shale phases of the formation are satisfactorily dense, strong, and impervious. The various phases of the Wolf Mountain formation appear to have adequate bearing strength.

The shale, including the silty phases, slakes in water after being dried. The slaking propensities vary considerably depending on its composition and texture. Therefore, it was necessary to observe the usual precautions against surface deterioration in all foundation excavations where concrete was placed.

The foregoing considerations indicate immediately the point of greatest danger; that is, the tendency of the shale to slake and form mud seams. One method of retarding this tendency during construction was to spray the exposed shale surface with an asphalt solution. This protective covering keeps the shale from drying and preserves its rock-like character. Since concrete must be placed on top of this protective coating, the degree of bond between the dam and foundation was unknown.

Laboratory Tests of Shale

Prior to making field tests, an extensive series of laboratory experiments were made with the shale at the Agricultural and Mechanical College at College Station, Tex. Additional tests were run at the Vicksburg Soils Laboratory conducted by the U. S. Army Engineers at Vicksburg, Miss. In general, the specimens were cut from cores taken by shot-drill methods during the exploratory boring and were carefully preserved until tested. The greater part of these tests were made on cylinders, 6 in. in diameter, 12 in. high, or on parts trimmed from the 6-in. cores. Although they varied over a wide range, the results of these tests were of a predictable nature and for the lower stresses indicated that the shale behaved as an elastic, isotropic material. The average results of the first tests conducted are as follows:

Test	Average results
(*a*) Moisture content, expressed as a percentage of the dry weight	7.85
(*b*) Percentage of total volume occupied by voids	15.4
(*c*) Absorption of water, expressed as a percentage of the total weight	0.25
(*d*) Modulus of elasticity, in pounds per square inch, at a compression of 7.2 tons per sq ft	133 000

Test	Average results
(*e*) Crushing strength of 6-in. by 12-in. cylinders, in tons per square foot	30
(*f*) Apparent specific gravity	2.46
(*g*) Weight per cubic foot undisturbed, in pounds	154
(*h*) Shearing parallel to bedding plane, in tons per square foot when compressed at 7.2 tons per sq ft	13.2
(*i*) Shearing normal to bedding plane, in tons per square foot when compressed at 3.6 tons per sq ft	8.35
(*j*) Chemical constituents, expressed as percentages of the total weight:	
Silica	62
Alumina	26
Lime	0.1
Magnesia	2.0
Potash and soda	some
(*k*) Cohesion, in tons per square foot (Vicksburg tests)	4.5
(*l*) Angle of internal friction, in degrees (Vicksburg tests)	45

Item (*c*) is not a standard or recognized test. The method used was to weigh specimens containing all of their natural moisture and then immerse them in water at room temperatures for periods varying from 3 to 20 days. It was noted that the specimens remained hard during this time and continued to absorb water for about 10 days. After the period of immersion the specimens were towel dried and re-weighed. The percentage of absorption was taken as the difference in the two weights, divided by the initial weight.

It is also to be noted that Items (*h*) and (*i*) fail to conform with tests (*k*) and (*l*) made at the Vicksburg laboratory. Disparity between results obtained by different laboratories, and even differences between several results obtained by the same laboratory, are the usual problems that confront a foundation engineer. A second series of tests covering essentially the same points gave slightly higher results and will be discussed subsequently under the comparison of laboratory and field experimental data.

In conducting the laboratory tests an attempt was made to duplicate the technique used in testing other earth, shale, and rock samples. New methods and refinement of apparatus were given some consideration but available time made it expedient to follow such rules as were known to have given good results in other cases. A further discussion of the laboratory tests is beyond the scope of this paper and they are mentioned merely to afford an over-all view of the material on which the field tests were conducted.

Sliding of Concrete Blocks on a Shale Foundation

On the right bank of the Brazos River the concave side of Possum Kingdom Bend has a gently sloping surface that formed an ideal place to dig a test trench down to shale. At the point chosen for the trench the elevation of shale was higher than low water in the Brazos River and all experiments were conducted without interference from the river or from inadequate drainage. Conglom-

erate and sandstone were excavated with the aid of a small power shovel and dump trucks, after drilling and blasting. When blue shale was reached, it was excavated to within 12 in. of final grade and an asphalt sealing solution was applied to the surface to keep the moisture within the shale from escaping. Immediately prior to casting the concrete blocks in place the last 12 in. of shale were removed by air tools and by hand, the surface was cleaned by an air jet and another sealing coat of asphalt solution was applied. The foundation for the test blocks had a 5° slope upward in the same direction as the direction of the horizontal force that was to be applied. This sloping contact plane enters into computations for the resultant load and the angle of friction between concrete and rock. The tangent of the angle of the resultant load on this 5° sloping plane from a line normal to it is the ratio of parallel to normal forces. The parallel and normal forces are the vectoral summation of proper components of the vertical and horizontal loads. No special effort was made to secure a sandpaper finish on the shale foundation and the small irregularities from the use of the air tools were believed to be comparable to the surface that would be obtained under the dam when it was constructed.

The concrete test blocks were made from local aggregates, river water and Portland cement. Frequent checks indicated an average weight of 153 lb per cu ft and concrete test cylinders made at the time the blocks were cast, as well as cores later drilled from the test blocks, showed an average crushing strength of a 6-in. (diameter) by 12-in. (high) cylinder to be appreciably more than 3 000 lb per sq in. Eight test blocks, measuring 5 ft by 5 ft by 5 ft were cast in place, but only three of these gave significant information about the shale foundation.

The apparatus for the tests consisted of vertical and horizontal hydraulic jacks for applying loads, and dial gages for measuring movements of the block. Two hydraulic jacks resting on the top of the block, with an overhead, ballasted steel platform for backing, transmitted a controlled and registered vertical force (see Fig. 1). The overhead platform spanned the test trench and rested on rollers placed on ledges or berms cut in the sandstone on either side of the trench about 10 ft above the foundation (see Fig. 2).

The rollers made the platform movable along the trench axis so that when one test was completed the entire platform was wedged over until it was directly over the next test block. In this manner considerable time was saved in erecting apparatus for the eight tests. The ballast on the platform was sand confined in bags.

Horizontal loads (see Fig. 3) were applied by five hydraulic jacks working against the base of the concrete test block and using the side of the trench for backing. Fig. 4 shows the apparatus in place. Shoring and blocking were necessary to distribute the loads properly to the shale backing. Jacks and hydraulic gages were calibrated after the tests were finished. Entire units were placed in a 300 000-lb testing machine, the load applied, and the gages were calibrated accurately.

Horizontal movement of the concrete blocks was measured by four dial gages at the lower four corners of the block. Vertical movement was measured by two dial gages at the front and rear on the center line of the block near the top. These dial gages were sensitive to movements of 0.001 in. It required some

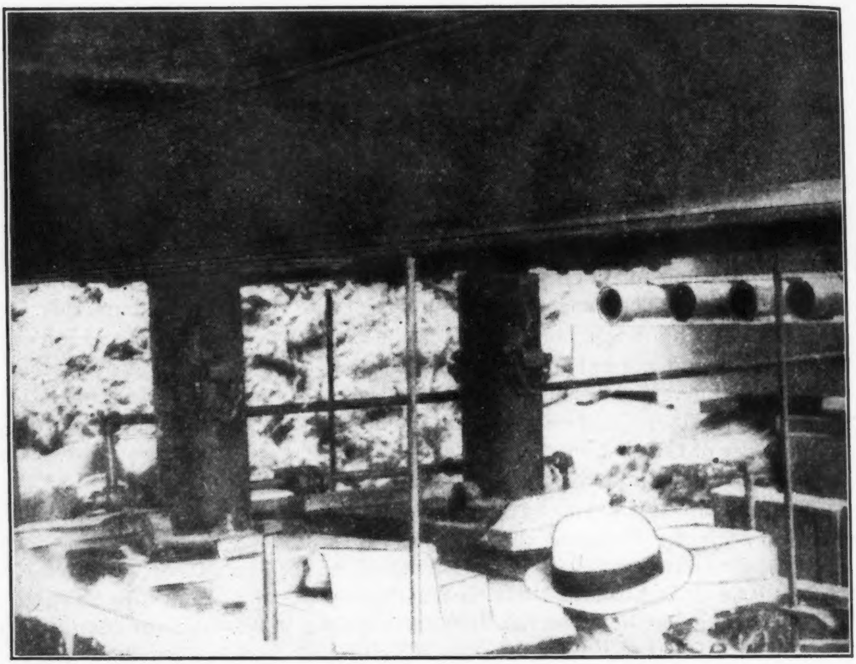

FIG. 1.—HYDRAULIC JACKS AND BLOCKING FOR VERTICAL LOAD APPLICATION

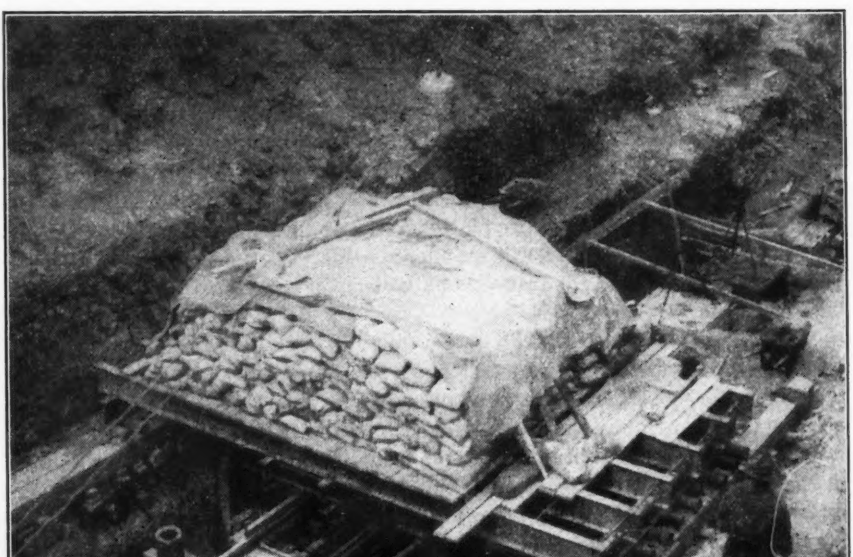

FIG. 2.—GENERAL VIEW OF THE TEST TRENCH AND APPARATUS

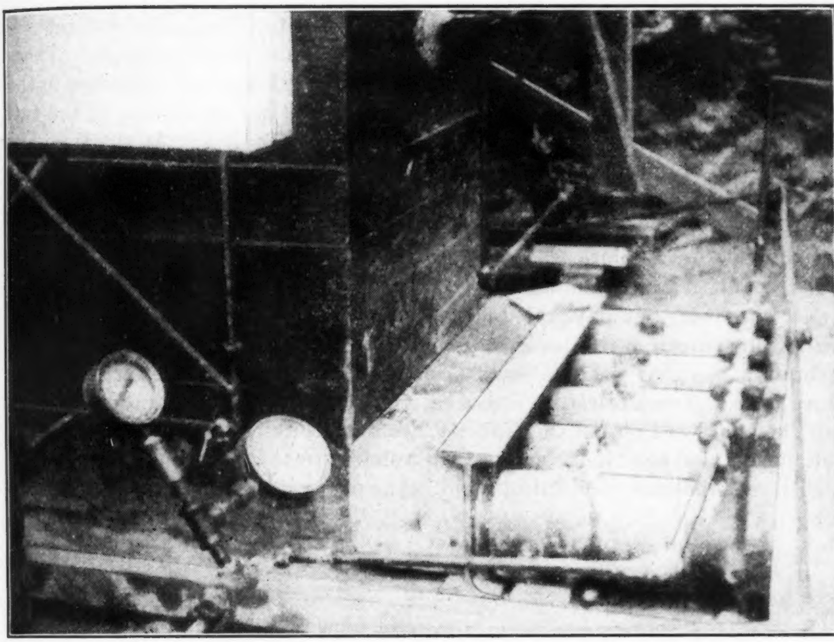

FIG. 3.—HYDRAULIC JACKS AND SHORING FOR HORIZONTAL LOAD APPLICATION

ingenuity to support these gages rigidly and independent from the test block. The framework finally used, as well as the relative positions of the six dial gages, is shown in Fig. 4.

The test procedure consisted of first applying the vertical load up to a predetermined value and then loading horizontally until the specimen failed. Movement of the block in a vertical and horizontal direction was recorded frequently during the loading. Time was kept from the beginning of the test until failure and a record was made at each reading interval to study the movement of the block.

TABLE 1.—FIELD OBSERVATIONS ON TEST BLOCKS

Test Block No.	Time, in minutes	Normal load, in tons per square foot	Settlement, in inches	Total horizontal load, in tons	Shear, in tons per square foot	Horizontal movement, in inches	Ratio of horizontal to vertical load	Ratio of parallel to normal load on foundation plane	Coefficient of friction	Percentage of base area in contact with shale*
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
2	24	0.94	0.004	35.15	1.31	0.072	1.49	1.302	0.857	15
3	34	2.66	0.026	69.5	2.52	0.193	1.04	0.879	0.848	17
8	47	3.17	0.089	72.0	2.63	0.22	0.904	0.77	....	100

* Percentage of the base area of the concrete test block to which shale was adhering after the test.

Data on the three blocks giving most information about the shale have been tabulated in Table 1. After failure of the specimens both loads were released

and then reapplied in an effort to get the coefficient of friction of concrete on shale, uninfluenced by bond between the monolith and the foundation. These results, with a record of the percentage area of shale found still adhering to the bottom of the blocks when they were tipped over at the conclusion of the two tests described, are given in Columns (9) and (10), Table 1. At the time the blocks failed (when no greater resistance to a horizontal load could be obtained) small cracks appeared in the shale immediately beneath the block. These small cracks, apparent at the bottom corners of the blocks in the shale, indicated local shear failure near the surface. The foundation for Block No. 8 was recognized as soft and inferior to that which ordinarily would be permitted under the dam; but physical difficulties in equipment and drainage made it inadvisable to excavate down to good shale. The fact that was given consideration was that this one poor specimen reduced the average stress for the three tests and would lead to conservative values for design purposes.

The ratios in Column (8), Table 1, are a measure of the angle that the resultant force on the blocks makes with a line normal to the foundation. The high values indicate mechanical bond between the concrete and shale prior to

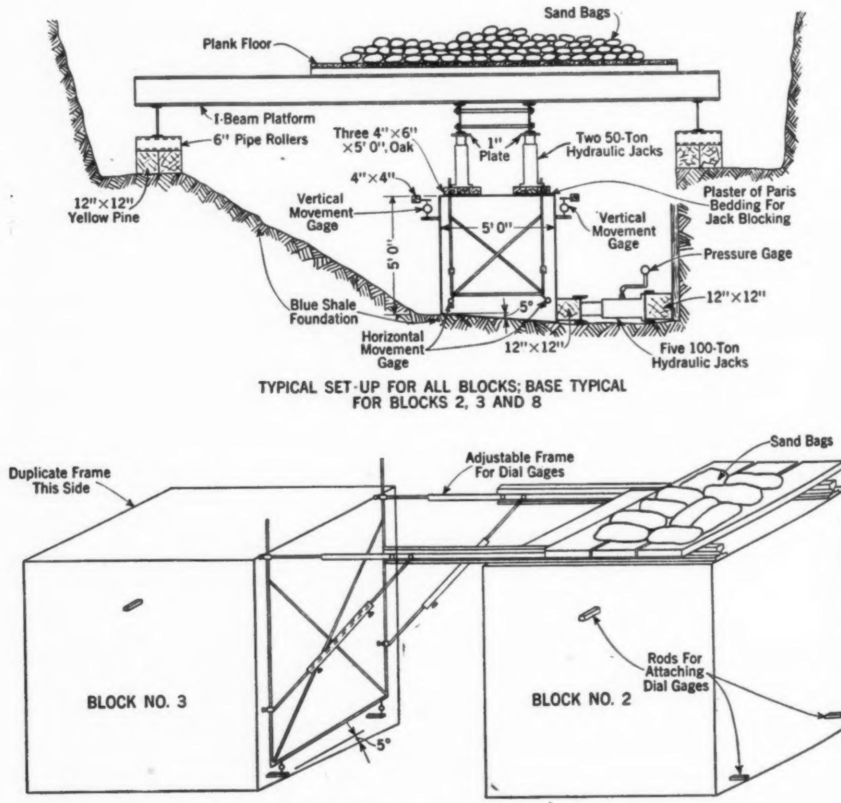

FIG. 4.—GENERAL ARRANGEMENT OF APPARATUS FOR FOUNDATION TESTS

failure. The shear values in Column (5), Table 1, are taken along the plane of contact between the shale and concrete. They show an expected increase with the increase in normal load. The coefficients of friction of concrete on shale (Column (9), Table 1) include the effect of bottom irregularities in the foundation caused by scars and depressions from the use of hand tools in excavating. Subsequently these irregularities were filled with concrete when the block was cast in place. Visual inspection of the bottoms of the test blocks,

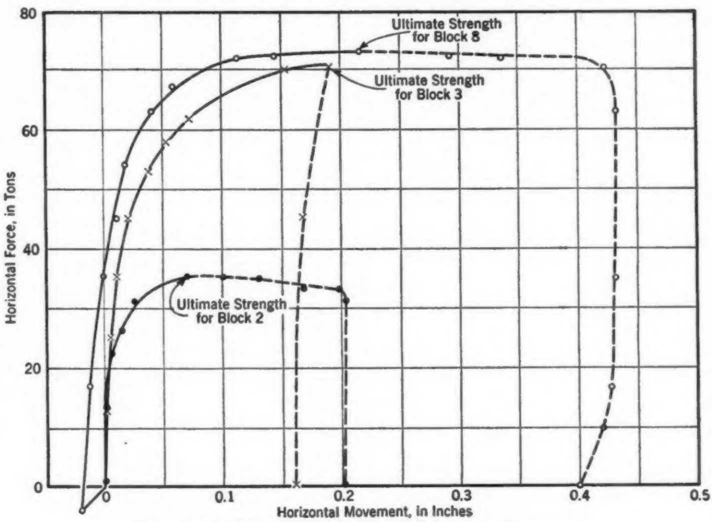

FIG. 5.—STRESS-STRAIN CURVES, BLOCKS 2, 3, AND 8

when they were turned over upon completion of the tests, indicated that local shearing of the shale during the second test gave this rather high value for coefficient of friction. The time involved in the tests was a matter of minutes and corresponded to a "quick tests." This factor must be evaluated properly, particularly in any extrapolation of these data. A graphical stress-strain relationship is given in Fig. 5, the vertical load being kept constant for each of the tests, as follows:

Test Block No.	Maximum vertical load, in tons
8	79.5
3	66.5
2	23.5

It should be noted in Fig. 5 that the specimens showed remarkable agreement with Hooke's law up to a fairly well defined point that might be termed the "yield point." This assumed yield point is about 63% of the ultimate strength and probably represents the first local failure in shear of a part of the shale immediately behind one of the protrusions of concrete that filled the irregularities of the surface of the foundation. The rate of increase in the ratio of horizontal load to vertical load was constant for all three specimens, to the point of ultimate strength.

The conclusions drawn from the foregoing data are:

(1) Bond existed between the concrete and shale in spite of the asphalt sealing solution that separated the two;

(2) The sealing solution preserved the rock-like qualities of the shale;

(3) If the dam is proportioned so that the ratio of the summation of horizontal forces to the summation of vertical forces does not exceed 0.50, a safe structure will result;

(4) Shear values are a function of vertical load and increase in direct proportion to the vertical load; and,

(5) Settlements were small for the loads applied but greater settlements should be anticipated for larger loads sustained for a longer period.

Fatigue Effect

The load on the shale foundation under the proposed dam would be subject to some variation depending upon the depth of water in the reservoir behind the dam. It was planned to store flood waters up to normal pool elevation and then release them gradually for domestic, irrigation, and power purposes down to a predetermined level established in the interests of conservation of water. Such operation would result in base pressures at a point on the toe of the dam that might vary from 10 tons per sq ft, maximum pressure, to 4.3 tons per sq ft, minimum pressure. The probable time involved for such a load cycle would be about one year and although no great concern was felt over such a condition, this aspect of the problem was deemed worthy of investigation.

A cast-iron plate, 12 in. by 12 in. by 1.5 in. thick, was carefully leveled upon the prepared shale foundation. Preparation of the foundation included final hand excavation of shale and a sprayed application of an asphalt emulsion. This protective coating was renewed at 20-day intervals or as often as small patches showed wear from foot traffic. The cast-iron plate was seated upon a very thin layer of Portland cement mortar to obtain a level, horizontal surface and to fill the minute irregularities on the rock surface.

Controlled and registered vertical loads were applied through an hydraulic jack backed against a bin full of sand resting on an overhead platform. To assure axial application of the load a spherical bearing block was provided between the jack and the loading platform. Settlement readings were taken by means of small dial instruments measuring to 0.001 in. at the four corners of the test plate and at four points on the shale 6 in. removed from the edges of the plate.

A load cycle was considered complete when the pressure had been increased slowly to 10 tons and then decreased to 4.3 tons. In general, one cycle would require about six weeks during which time daily observations were taken of settlement, temperature, and weather. Some difficulty was experienced at first but corrections were soon made and the value of the test was not lost. Four cycles were completed in 246 days and the graphical record is indicated in Fig. 6.

In general, the shale behaved as an elastic material that would compress upon load application and rebound when the load was released during the first two cycles. The greatest average settlement of the four corners of the plate

was 0.07 in., and 0.02 in. for the points on the shale 6 in. removed from the plate edge. The data are decidedly irregular and cannot logically be plotted as a stress-strain graph. In a gross manner it can be stated that there is a

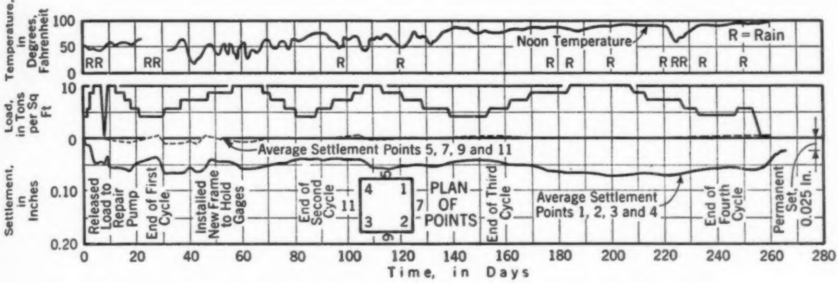

FIG. 6.—CYCLIC LOADING AND SETTLEMENTS, 12-INCH BY 12-INCH PLATE

tendency toward elastic hysteresis since the "loading" line of a stress-strain graph does not coincide with the "unloading" line. This phenomenon was more apparent in laboratory tests of cyclic loading. A second observation is that the deformation at the end of the fourth cycle was more than that obtained in the first, second, and third cycles. There is a tendency for the settlement curve to flatten and become horizontal after repeated loadings, indicating some change in the structure of the rock. The permanent set of the area immediately under the test plate, and the slight upheaval of the points 6 in. removed from the plate, is typical of compaction and the probability of some plastic flow. However, the magnitudes of these movements of the foundation are small and it remains a matter of conjecture as to the number of loading cycles before definite fatigue effects could be noted. Certainly, for the loads involved in the test applied at the same rate for four cycles, the settlements are without structural significance. There is a recognized danger in assuming that a single test result on a specific, small area of rock can be used indiscriminately on the entire foundation. The difference between areas under test and under the prototype is too great to use the results of the experiment as any thing more than a trend or indication.

TIME EFFECT ON SETTLEMENT

The time effect on settlement was investigated from the results of loading three cast-iron test plates and observing the settlement. Since the preparation and test procedure for obtaining the time effect is the same for all three plates, a description of only the 12-in. by 12-in. by 2-in. plate will be given. The final results will include data on all three plates. The dimensions of the other two plates were 6 in. by 6 in. by 2 in., and 24 in. by 24 in. by 2 in., respectively.

A registered and controlled vertical force was transmitted to the 12-in. by 12-in. plate through an hydraulic jack. The jack used the overhead loaded platform previously described, for backing, and axial application of the load was insured by a spherical bearing block between the jack and the platform. A refinement over the previous equipment consisted of a pressure regulator

and check value inserted in the pipe line between the pump and the jack. The pressure regulator gave constant pressure with a minimum of manual attention, and the check valve permitted withdrawal of the pump from the line without loss of line pressure. If repairs to the pump were necessary, the accumulated test data would not be lost. The test plate itself was leveled carefully and set upon the prepared shale foundation in the same manner as the one used for the cyclic loading test. Settlement readings were taken on the four corners of the cast-iron plate and on the shale at a distance of 12 in. from each of the four edges of the plate. A graphical record of all recorded data appears as Fig. 7.

Equilibrium was reached only by the smallest test plate after 60 days of constant load application. No further settlement was noted although the load was kept on the plate for 121 days. The medium size, or 12-in. by 12-in. plate, showed a decreasing rate of settlement with some indication of a "sec-

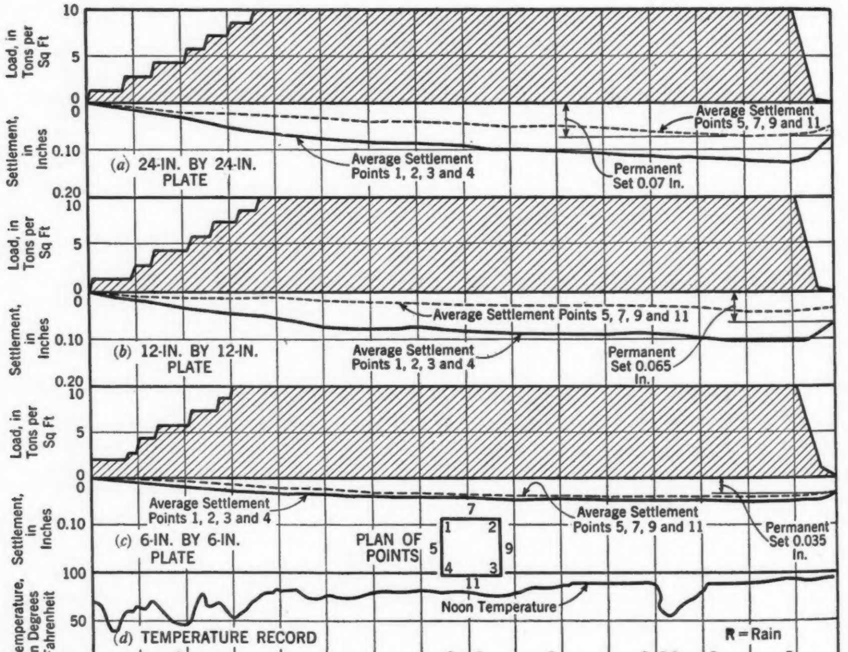

FIG. 7.—SETTLEMENT OBSERVATIONS UNDER LOADS OF TEN TONS PER SQUARE FOOT

ondary" time effect after 70 days of constant load application. The large 24-in. by 24-in. plate continued to settle at a slow, uniform rate after the first 30 days of sustained load. The first 30 days was a period of decreasing rate of settlement.

Fig. 8 shows settlement plotted against time on a semi-logarithmic scale for the three plates. A smooth curve is drawn, based upon these points, and the difficulty of making simple, comprehensive statements from these test results is evident immediately. The three curves do not agree in shape. Although a general agreement of form can be said to exist for the time settlement curves for the medium and large plates, the data on the smallest plate do not give the same shape of curve. The conclusion drawn is that the rate of settlement is a function of the loaded area and the total load. Furthermore, the rate is slow enough (0.01 in. in 20 days for the largest plate) so that in the prototype the stability of the structure will not be affected. Continued settlement of the dam over a long period should be anticipated.

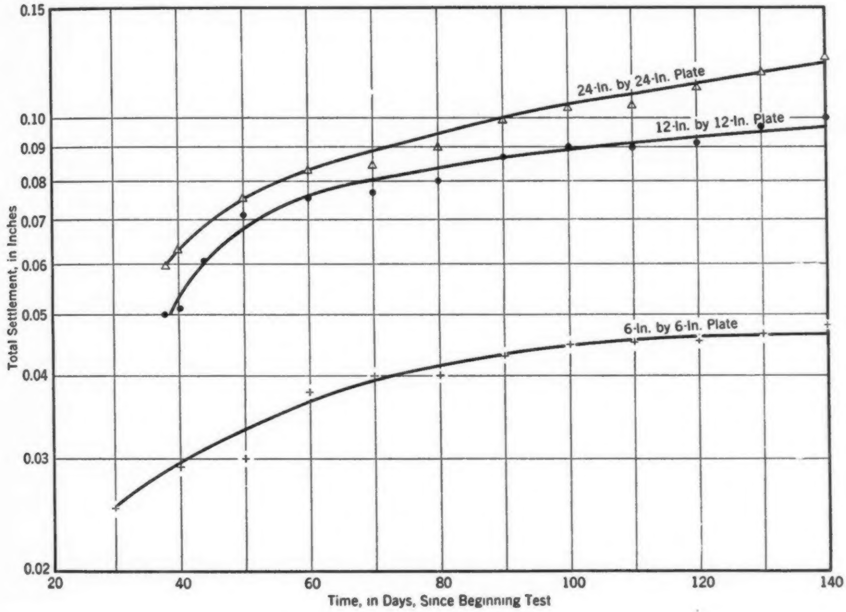

FIG. 8.—TIME-SETTLEMENT CURVES

The total settlement of the plates in all three cases was greater than the shale settlement 12 in. from the edge of the plates. No bulge in the surrounding shale was observed. This dish-shaped condition under load suggests that Boussinesq's formula, modified by a parameter, may be applicable. However, the Poisson's ratio of the shale is not known and a check on the foregoing supposition is precluded. No drift or plastic flow was discovered and settlement was a function of compaction only.

It has been stated[2] that the amount of settlement in soil is equal to the product of a constant depending upon the character of the soil, the diameter of the loaded area, and the unit load on the area. The tests on the three different sizes of square plates, each loaded with the same unit load, and founded on the

same type of material, form the basis for establishing the relationship between the size of the loaded area and settlement. The equipment and tests are identical with that described under the heading "Time Effect on Settlement."

The results of these tests, after 116 days under a constant unit load of 10 tons per sq ft, showed a settlement for the smallest plate of 0.048 in. and an average settlement of the shale, 12 in. removed from the plate, of 0.044 in. The medium size plate having four times the area settled only 0.10 in. and the shale 12 in. away settled 0.036 in. The largest plate with an area of 4 sq ft settled only 0.123 in. and the outside shale, 12 in. away, settled 0.072 in. These values are of an expected order and can be used in the formula

$$S = C\,D\,w \quad \dots\dots\dots\dots\dots\dots (1)$$

in which: S = settlement, in inches; C = constant = 0.08; D = diameter or side of plate, in feet; and, w = load per square foot = 10 tons.

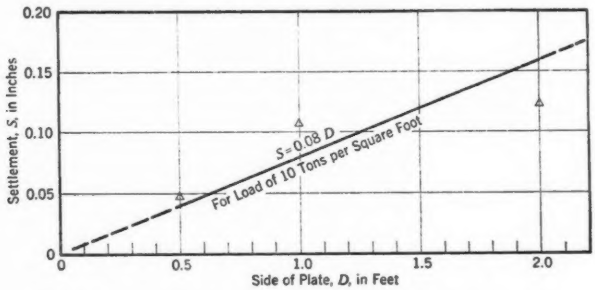

FIG. 9.—RELATION BETWEEN TEST RESULTS AND EQUATION (1)

Fig. 9 shows this relationship between experimental results and Equation (1) graphically; the straight-line variation may be assumed without too great a loss of accuracy. The deviations represent unexplainable phenomena that frequently crop up in foundation experiments in spite of painstaking efforts to insure comparable results.

Fig. 7 indicates that the permanent set under the 6-in. by 6-in. plate was 0.035 in. upon load release and 0.065 in. and 0.70 in., respectively, for the 12-in. by 12-in. plate and 24-in. by 24-in. plate when the load was released. This variation points to more compaction under the larger area and suggests that not only is the settlement under load a function of size but that the permanent set is likewise governed by the size of the loaded area.

The dish-shaped surface of the rock in the immediate vicinity of the three test plates indicates a cohesive medium offering resistance to shear. Since the shale around the smaller plate settled more than that around the 12-in. by 12-in. plate, it follows that there is some "edge effect" or disturbance. W. S. Housel,[3]

[2] "The Science of Foundations—Its Present and Future," by Charles Terzaghi, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 270.

[3] "Notes on Soil Mechanics and Foundations," by Fred L. Plummer, M. Am. Soc. C. E., Edwards Brothers, Inc., Ann Arbor, Mich., 1936.

M. Am. Soc. C. E., has taken this factor into account in one of his older formulas for cohesive soils:

$$P = A\,m + l\,n \quad \text{.......................... (2)}$$

in which: P = total allowable pressure in tons on footing; A = area of footing, in square feet; l = perimeter of footing in linear feet; and, m and n = two distinct constants. The constants can be established by considering the load causing equal settlement on two of the plates and checking against the third plate load for a similar settlement. This has been done for a settlement of 0.05 in. and the computed constants are $m = 6.2$ and $n = 0.525$. The agreement for other equal settlements is rather poor using these exact constants but there is no reasonable doubt that bearing capacity is a function of area and perimeter.

The conclusions that may be made from the experiments on loading and settlement of test plates on this shale foundation are:

(1) Cyclic loading of the foundation is not structurally important (the suspected plastic flow of the shale under repeated loads is small and at a slow rate);

(2) Continued settlement of the dam over a long interval at a slow rate can be expected logically;

(3) Careful proportioning of footings so that the larger footings carry smaller loads per square unit will prevent differential settlement; and,

(4) Edge effect of footings are important as well as the size of the loaded area.

Correlation of Laboratory and Field Tests

Rapid laboratory tests were made to determine shear and sliding values on small samples of shale 6 in. in diameter and about 2.5 in. high, on which was cast a neat sand-cement mortar cylinder of the same diameter and about the same height. The data in Table 2 can be compared with similar material for field tests on Blocks Nos. 2, 3, and 8, in Table 1. If the normal load is plotted

TABLE 2.—Laboratory Observations on Test Blocks

Item No.	Description	Normal Load, in Tons per Square Foot		
		10.8	7.2	3.6
1	Number of specimens	3	3	1
2	Average shear, in tons per square foot	12.46	10.8	6.9
3	Ratio of horizontal load to vertical load	1.15	1.50	1.91

against average shear (Item No. 2, Table 2), the resulting graph conforms to the equation

$$x = 3.7 + 0.9\,y \quad \text{.......................... (3)}$$

One reason for the higher laboratory test values is that there was no asphalt coating separating the concrete and the shale. This is reflected by the lower bond or shear values for the field tests, amounting to an 81% reduction. Sliding friction was also lower by 28% for the field tests, for the same reason.

At the Vicksburg laboratory, three samples of shale were sandpapered smooth, coated with an asphalt sealing solution and a neat sand-cement mortar block was cast upon the top of the shale sample. The specimens were placed in a shearing apparatus with a known load normal to the plane of contact of the two materials, and then a load parallel to the contact plane was applied. After the slight bond was broken the coefficient of friction of the materials was 0.506. This value is 31% lower than that obtained by the field tests and gives a clear idea of the effect of irregularities in the contact plane in the field tests.

The shale was subjected to cyclic loading, at an accelerated rate, in the laboratory. A 6-in. by 12-in. cylinder of shale was placed in the center of a 16-gage corrugated iron culvert pipe 13.5 in. in diameter and 12 in. high. The annular space between the shale and the pipe was then filled with plaster of Paris. Load on the shale area 6 in. in diameter was applied through a disk measuring 4.91 in. in diameter. Compaction was measured by Ames gages. Two tests, for 16 and 24 cycles, respectively, varying from no load to 15.2 tons per sq ft gave a deformation of 0.006 in. and 0.0036 in. after four cycles. After sixteen cycles the permanent set was 0.0084 in. and 0.006 in. Elastic hysteresis and elastic after effects were both noted in these experiments. Likewise, it was apparent that the minimum deformation occurred under rapid application of load and an early release of that load. Since the conditions of the laboratory tests were totally different from the field tests no comparison is possible.

Several other unrelated facts were noted in the laboratory experiments, all of which defy comparison with the field tests. One of these was the 100% consolidation of a small shale specimen under a load of 3.576 tons per sq ft. Furthermore, by crushing many 6-in. cylinders of shale, 12 in. high, a general increase in crushing strength was noted for increase in depth at which the specimens were taken. This same relationship held for resistance to shear. Although this information "puts the final touches" to the pictured problem, greatest confidence is felt in the results of the field tests. Whatever else may be claimed about the field tests, it is undisputed that errors incident to sampling and to duplication of natural conditions are eliminated. It is believed that, in any comprehensive foundation investigation, the test results obtained in the laboratory should be supplemented by those taken in the field.

Loads Used in the Design of the Dam

Detailed plans were drawn for a gravity dam and a round-head buttress dam for the same site. The field tests indicated a safe bearing capacity of 10 tons per sq ft, assuming correct proportioning of footings. The maximum unit loads used in the design were 9.1 tons per sq ft for the gravity type of dam and 9.8 tons per sq ft for the round-head buttress type. The maximum average shear resistance on the plane of contact of these two types, with the foundation, was 2.3 and 3.6 tons per sq ft, respectively. This compares favorably with the field tests of 5.1 tons per sq ft under a much smaller vertical force. The ratio of the horizontal to the vertical forces used in design was 0.332 for the gravity dam and 0.486 for the round-head buttress dam. This is about six-tenths of the ultimate values from the field tests, 0.812.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THE YELLOW RIVER PROBLEM

Discussion

BY GEORGE HIGGINS, M. AM. SOC. C. E.

GEORGE HIGGINS,[31] M. AM. SOC. C. E. (by letter).[31a]—Valuable material and suggestions have been furnished by the authors. The writer suggests that the entire length of that part of the river in which the flow is above the level of the adjacent country shall have its banks constructed to act as spillways. This means a spillway on the left bank about 465 miles long, and one on the right bank about 450 miles long. These are approximately the lengths of the dikes or levees already constructed. It is suggested that, in general, a shallow flow of flood waters over the adjacent country on both sides of the river would benefit the land, which sometimes suffers from drought, and would enrich it with the silt derived from the uplands. Not only that, but the relief from high flood conditions which the river would experience in consequence of the diversion of so great a proportion of the flood waters would greatly lessen the danger of concentrated flood waters which now sometimes break through the levees.

In effect, the proposal is to follow Nature's procedure. Heavy rains wash material from the hill country into the valleys where much of the mixture of solids and liquid is carried along. Sooner or later, heavier particles will settle on the stream beds, raising them; and, when these beds are high enough and the banks low enough, overflow will occur. Generally such overflow is accompanied by deposition of silt on the land adjacent to the stream, heavier and coarser particles lodging close to the stream, the finer being carried on. This accounts for the form which cross-sections present—gradual surface slopes falling away from the relatively higher river banks. Rivers in all parts of the world exhibit this natural action.

On the other hand, if it is attempted to confine flood flows to a definite channel by erecting artificial banks, the bed will continue to rise in consequence of the subsidence of the heavier particles which the water is carrying forward—this is the ultimate result even if, at times, the action of the current may be a

NOTE.—This paper by O. J. Todd, M. Am. Soc. C. E., and S. Eliassen, Assoc. M. Am. Soc. C. E., was published in December, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1939, by Messrs. J. W. Beardsley, and Elliott J. Dent; April, 1939, by Messrs. Herbert Chatley, and H. van der Veen; and June, 1939, by Messrs. C. S. Jarvis, and E. W. Lane.

[31] Cons. Engr., Melbourne, Victoria, Australia.

[31a] Received by the Secretary June 16, 1939.

scouring one. One of the most notable examples of this effect is furnished by the Po River in Italy, whose banks have had to be raised several times to keep pace with the higher water level due to the raising of the river-bed. About 5 000 000 persons live on areas protected by these banks, and, when the banks break through (as they have done, unfortunately), the results are disastrous.

"China's Sorrow," the Yellow River, carries an enormously greater proportion of sediment than the Po, and the fact that much of this sediment is deposited on the river-bed as a result of floods is well known. Were it not that the artificial banks have frequently broken through, the rate at which the bed rises would be still more rapid. What will happen if the levees are made unbreakable? The channel will fill up, and, if the banks are raised higher, the channel filling will continue. Terrible as the results of bank fracture have been in the past, they will be worse if still higher banks break through. In any case, there must be a limit to this bank building.

The simpler plan seems to be to strengthen the river's natural banks so as to make them suitable to act as spillways, and to let the river deposit silt on them and outside them at its natural slope—probably not steeper than 6 in. to the mile. To a great extent, the river itself will raise its own banks as well as its bed. Man may help, here, to direct the natural action: By strengthening the natural banks; by raising them in places so as to keep the flow over the crests approximately uniform in depth; by checking any tendency for the flow over the country's surface to concentrate itself in depressions and cause scour there; by building groins or other means for deflecting the river's current away from the bank which it may tend to undermine; by encouraging the growth of vegetation on the loess and other surfaces from which the silt is derived; by interposing positive checks to the flow of silt from eroding surfaces; and, generally, by working with Nature.

Such details as the raising of dwellings and providing for the safety of live stock during the month or two that the shallow flooded conditions may last may well be left to the resourceful people who have displayed remarkable skill and endurance in the past.

In the writer's opinion, the existing levees should be removed. If they remained, they would only obstruct the even flow of silt-laden water over the surface of the country; and this flow is desired. The material in the levees would be useful, in places, for the strengthening of the river banks, thus enabling them to act as spillways, and also for forming groins or training-walls in the channel.

As for the height of the banks, which would become spillway crests, the writer suggests taking present bank levels, where they have not been altered artificially, as a guide at the outset. Absolute uniformity of depth flowing over the crests will be unattainable. Local falls of rain would prevent such uniformity, producing "flood waves," as they do. Moreover, every large flood has its "peak," and the peak passes down stream while gradually flattening itself out.

Earthquakes are known to affect the country through which the Yellow River flows. Any shattering due to this cause will be much less serious with river banks of indefinite width than with banks confined by comparatively steep

levees. Ice jams may be expected to occur in the future, as in the past, and the consequent "heaping up" of water locally will have to be accepted.

In common with all irrigation systems, there is a possibility of the irrigated land becoming alkaline, necessitating drainage, which is not a simple matter in country so nearly level as that here considered; but such difficulties, and probably many others, will have to be faced. The aim must be to eliminate difficulties as far as possible.

The growth of the delta has occasioned anxiety to many engineers who have studied the Yellow River problem. This growth would be greatly lessened if much of the silt were deposited on the country adjacent to the river. The "plains" through which the Yellow River flows for the last 500 miles of its course have evidently been deposited as deltas. Allowing Nature to continue its processes should not lead to conditions inferior to those existing before Man interfered. In any case, even if it were possible to form and maintain a channel between levees which will not overflow, it will not be possible to discharge all the silt into the ocean. A certain quantity will be deposited on the river-bed, making it necessary to raise the levees frequently. Where would that end? It would create a channel on top of a steep embankment, rising higher and higher as time goes on! There is no satisfactory end to this course of action.

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DISCUSSIONS

THE RISK OF THE UNEXPECTED IN SUB-SURFACE CONSTRUCTION CONTRACTS

Discussion

BY CARROLL A. FARWELL, M. AM. SOC. C. E.

CARROLL A. FARWELL,[15] M. AM. SOC. C. E. (by letter).[15a]—A subject that has needed more study is presented in this paper, and its treatment by the author is of much value. The writer wishes to add emphasis to the importance of securing, in so far as possible, a definite "meeting of the minds" of the contracting parties. Each party must comprehend that there are risks to be taken and that the contract documents should allocate these risks clearly.

It appears that, in general, the Courts uphold the legality of placing a risk upon the contractor provided that the contract documents so state. Certain risks, which may not be mentioned specifically in the contract, are customarily assumed by the owner—such as, for example, extremely abnormal weather conditions or river floods, or hurricanes (in New England). Similarly, variations in the cost of materials and labor are customarily borne by the contractor. It would be better if the allocation of these risks were definitely specified instead of being left in the classification of "common practice." The risk of fire may be carried by either party and the allocation of this risk is often clearly defined in the contract through a specification that the contractor, or the owner, carry fire insurance. The risk involved in sub-surface conditions, to which the author's paper is limited, is one of the major items of construction work and, as such, needs accurate allocation.

Manual No. 8 states (1(*b*))[2]

> "* * * The owner should provide the necessary funds for a complete preliminary investigation of the soil, its geological formation, and water conditions; and for the determination of all other facts needed to enable

NOTE.—This paper by Oren Clive Herwitz, Esq., was published in January, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1939, by Messrs. Alonzo J. Hammond, Frederick W. Newton, David A. Molitor, F. B. Marsh, and Evan S. Martin; May, 1939, by Messrs. C. Maxwell Stanley, and Francis J. Morgan and Frederick C. Zeigler; and June, 1939, by Messrs. Lazarus White, and T. Kennard Thomson.

[15] Cons. Engr. (Fay, Spofford and Thorndike), Boston, Mass.

[15a] Received by the Secretary July 25, 1939.

[2] For reference to numerals in parentheses, see "Court Citations and References," in the Appendix [at the end of the paper].

> the engineer to design the work properly and the contractor to determine its cost accurately * * *. It is the contractor's duty to estimate only on work for which adequate preliminary information has been obtained, and for which adequate designs and specifications have been prepared by the engineer. He should not bid on the work until he has assured himself as to the adequacy of the information given him and the practicability of the type of construction shown in the engineer's design * * *.

The words "accurately," "adequacy," etc., are relative and their interpretation involves the personal equation. Regardless of how accurate and how adequate is the information given, there is a possibility of unforeseen conditions affecting the cost of the work, and the contract documents should specify which party is to bear, or is to be benefited by, the change in cost. Moreover, in order that the engineer may determine what are unforeseen conditions, the foreseen conditions must be shown or indicated definitely.

As has been stated, the contracting business is highly competitive and the writer agrees that the more the risks are carried by the owner, the lower will be the contract prices bid, and the more stable the contracting business. On the other hand, it is sometimes true that the owner is also engaged in a highly competitive undertaking, as for example, in the construction of an office building in which the cost of the building affects the rents that must be obtained; and, if the cost is greater than a certain limit, the project is not financially feasible.

Therefore, the owner may consider it desirable to carry as little risk as possible himself and to employ the contractor to carry all possible risks in order that he may know his upset cost as accurately as possible. This will usually cause the work of the contract to cost more than would otherwise be the case; but the additional cost may be well worth while. Moreover, the risk must be assumed by some one, even if it does not appear in the cost figures. If it were possible to arrange for some kind of insurance to cover such a risk (as is done by insurance against fire), the owner would undoubtedly be willing to pay for it, either directly or through the contractor.

If it were possible to include an item in the contract entitled "Assumption of Risk of Unexpected Sub-Surface Conditions," it would be highly desirable as it would call attention to the actual cost of carrying such a risk.

Public bodies, as well as private owners, often desire to know the upset cost of a project as nearly as possible in order to assure themselves that the cost will not exceed the funds available; and here again the contractor may properly be paid for the risk which he assumes.

The owner expects the engineer's estimate of cost to be reasonably accurate, and the Engineer should state (if it is a fact) that there are certain risks which may increase that cost over the estimated contract cost. This is often done under an item called "Contingencies" but many times this item is removed from the estimate when the contract documents are executed.

Under certain conditions, it has been the custom in the writer's office to arrange construction contracts for bridge foundations so that the contractor will be paid a lump-sum price for the foundations as built in accordance with the contract plans, and be paid under another item a unit price per vertical foot for the additional depth to which any pier may be carried below the depth

indicated on the contract plans. This has been done on the theory that a decrease in depth was unlikely and, moreover, would have little effect upon the total contract cost, whereas each foot of additional depth would increase the cost materially.

As an example of an apparent lack of clarity in the "meeting of the minds" in a contract for foundations, the following, which came under the close observation of the writer, may be of interest.

The Contract Documents included the following:

> "ART. 3. *Changes.*—The contracting officer may at any time, by a written order, and without notice to the sureties, make changes in the drawings and (or) specifications of this contract and within the general scope thereof. If such changes cause an increase or decrease in the amount due under this contract, or in the time required for its performance, an equitable adjustment shall be made and the contract shall be modified in writing accordingly * * *.
>
> "ART. 4. *Changed conditions.*—Should the contractor encounter, or the Government discover during the progress of the work, sub-surface and (or) latent conditions at the site materially differing from those shown on the drawings or indicated in the specifications, the attention of the contracting officer shall be called immediately to such conditions before they are disturbed. The contracting officer shall thereupon promptly investigate the conditions, and if he finds that they materially differ from those shown on the drawings or indicated in the specifications, he shall at once, with the written approval of the head of the department or his representative, make such changes in the drawings and (or) specifications as he may find necessary, and any increase or decrease of cost and (or) difference in time resulting from such changes shall be adjusted as provided in article 3 of this contract."

The specifications included the following:

> "*Borings.*—Wash borings have been made under the direction of the contracting officer and of the engineer at various points at the sites of the work. These borings were made in the usual manner and with reasonable care; and their locations, depths, and the character of the material apparently encountered have been recorded in good faith on the contract plans. Samples of material obtained from the borings have been preserved and labeled and may be examined at * * * . There is no expressed or implied agreement that the depths or the character of the material have been correctly indicated and bidders should take into account the possibility that conditions affecting the cost or quantities of work to be done may differ from those indicated.
>
> "*Boulders.*—The borings apparently indicate that boulders may be encountered in varying numbers and sizes at any or all of the excavations to be made for the work."

During, and at the completion (1934) of, the work the contractor claimed, in effect: That the number of boulders and the quality of the sand encountered in the excavation was materially different from that shown in the drawings and indicated in the specifications; that, therefore, unforeseen and unanticipated sub-surface and latent conditions were actually encountered and it was necessary to change the method of work; and, that adjustments of the contract sum and of the contract time should be made to compensate for the extra work and time resulting from such changes. The controversy was finally taken to the Court of Claims and, in so far as is known to the writer, decision is still (1939) pending.

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DISCUSSIONS

EARTHQUAKES AND STRUCTURES

Discussion

By Henry D. Dewell, M. Am. Soc. C. E.

Henry D. Dewell,[29] M. Am. Soc. C. E. (by letter).[29a]—This paper is an excellent and terse presentation of those fundamentals of the behavior of earthquakes, and of structures during earthquakes that are of particular importance to the structural engineer. In Section I, Mr. Galloway has quite properly emphasized the complexity of the earthquake and of the problem of giving a structure earthquake resistance.

The writer believes that the statement (see heading: "Earthquakes and Their Effects: Relation to Structures"), "In the case of high, narrow structures the vibration period is often close to that of some of the maximum waves and, unless the structure is well designed, serious damage will result if the shock is of long duration," should be amplified. The danger of resonance is not confined to high and narrow structures; neither is it at all certain that only the maximum earthquake waves are dangerous to structures. Herein it is understood that by the term "maximum earthquake waves" is meant the transverse waves that have the greater amplitudes. If the periods of the earthquake and the building are substantially in agreement, serious damage will undoubtedly occur, even if the shock is of short duration; consequently even a "well designed" structure will probably suffer seriously. After all, the term "well designed," when used in connection with ability to resist an earthquake, is rather relative. The writer understands that, as used, it means a construction with the structural units well arranged, designed for ample dead loads and live loads, with conservative unit stresses, with walls built integral with the structural frame or well tied and anchored thereto, and with all connections of sufficient capacity to develop the full strength of the members connected; and, in addition, it means that the structure has been designed to resist a definite lateral force from any direction.

Note.—This paper by Leander M. Hoskins, Esq., and John D. Galloway, M. Am. Soc. C. E., was published in December, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1939, by Messrs. Homer M. Hadley, and R. McC. Beanfield; April, 1939, by Messrs. R. S. Chew, Jacob J. Creskoff, and Arthur C. Ruge; May, 1939, by Walter L. Huber, M. Am. Soc. C. E.; and June, 1939, by A. A. Eremin, Assoc. M. Am. Soc. C. E.

[29] Cons. Civ. Engr. (Dewell and Earl), San Francisco, Calif.

[29a] Received by the Secretary June 16, 1939.

Mr. Galloway has correctly stated (see heading "Earthquakes and Their Effects") that, "The foundation material is the important element of the movement [of the ground in an earthquake]. Solid rock moves the least; deep alluvial soils saturated with water * * * will have the greatest movement." Lest inference be drawn from this statement that damage is always greater when the structures are founded in alluvial soils than in harder or diluvial soils, it may be mentioned that in the 1923 Tokyo earthquake the damage to brick and concrete buildings was greater to those buildings situated on the up-town diluvial soil than to similar buildings situated in the down-town soft alluvial soil.[30] R. R. Martel, M. Am. Soc. C. E., states[31] that:

> "* * * somewhat similar results were obtained from a study of effects of the Long Beach earthquake of 1933. There the damage to brick buildings located on the softer, more recently deposited alluvium with ground water from 2 to 10 ft. from the surface, was less than to similar buildings on the older, more firmly consolidated marine terrace deposits with ground water not so close to the surface."

Referring to the relative accelerations in soft and hard grounds, he aptly states, "for one earthquake the flip of a coin will furnish as satisfactory a guide as any in forecasting the relative accelerations on different materials." The foregoing statements are illustrations of the often observed phenomenon, that almost every rule or law of earthquake action on structures has some exceptions which are apparently inexplicable.

Referring to the possibility of resonance, Professor Hoskins states (heading "Conclusions from General Theory of Forced and Free Oscillations") that to prevent this

> "* * * it would be necessary to have full knowledge * * * regarding free-oscillation frequencies of actual buildings * * *.
>
> "As regards the second question, it is obvious that the designer is powerless to avoid the dangerous condition unless he is able to estimate in advance the natural oscillation frequency of any projected building. Of great value for this purpose would be data concerning actual oscillation periods of many existing structures so selected as to serve as types."

In this connection Mr. Galloway has noted (heading: "Earthquakes and Their Effects") that "The Earthquake Committee filed with its report[3] the vibration periods of a number of buildings in San Francisco * * *. Other data of similar kind have been published." It may be well to amplify Mr. Galloway's statement by noting that from this rather small beginning of the Earthquake Committee resulted the extensive program of measurements of periods of buildings, bridge piers, water tanks, and dams, by the U. S. Coast and Geodetic Survey, the results of which have been published in the *Bulletins* of the Seismological Society of America and are summarized in the Survey's

[30] "The Vibration of Structures and a Method of Measuring It," by Kyoji Suyehiro, *Inokuty Technical Papers*, 1928.

[31] "Effect on Foundation of Earthquake Motion," by R. R. Martel; presented at the meeting of the Soil Mechanics and Foundations Division, San Francisco, Calif., July 26, 1939 (not published).

[3] Unpublished Report of the Special Committee of the American Society of Civil Engineers on Effects of Earthquakes on Engineering Structures with Special Reference to the Japanese Earthquake of September 1, 1923. Available for reference at Engineering Societies Library, 33 West 39th Street, New York, N. Y.

Special Publication No. 201, "Earthquake Investigations in California, 1934–1935." In all, the Survey made about 1 200 observations on 400 buildings, 150 observations on 41 tanks, 200 observations on special structures, and 500 ground observations. A portable ground and building shaker was developed at Stanford University, Stanford University, Calif., and tests were made with this machine on buildings, dams, and bridges, and on the foundation ground itself.

Because of the complexity, intricacy, and the length of mathematical work necessary for the solution of the stresses in any but the most elementary forms of structures when subjected to even simplified types of vibratory motion, the writer believes that the greatest gain in current knowledge of earthquake-resistant design will come from experimental studies of models of buildings and other engineering structures subjected to artificial vibration of predetermined pattern, amplified by theoretical study. The most promising procedure is the determination, if possible, of those static loads which may be used as the equivalent of the dynamic earthquake shears for buildings of different shapes, heights, and mass distribution. Valuable work in this field has already been done by Prof. Lydik S. Jacobsen in the Vibration Research Laboratory at Stanford University, Professor Martel at the California Institute of Technology, at Pasadena, Calif., and Prof. Arthur C. Ruge at the Massachusetts Institute of Technology, at Cambridge, Mass. Some of the results of their work have been published from time to time in the technical press. Of particular interest are the experiments[32] of Professor Jacobsen on a model of a 16-story and a 29-story building.

A special type of structure studied by the writer, acting as a member of Mr. Galloway's Earthquake Committee,[33] was the tall, free-standing chimney. Sufficient data were available, through the courtesy of the Japanese members of the Committee, on the behavior of a large number of these chimneys during the Tokyo earthquake of September 1923, to warrant a study. The writer endeavored to correlate the location of the fractures in these chimneys with the theoretical locations when computed according to the impact theory presented by Professor Le Conte[27] in 1926 using assumed ground accelerations thought to be reasonable. Conversely, he computed the ground accelerations theoretically necessary to cause the fractures that had occurred. In these computations the formulas developed by Professor Le Conte were modified with the assistance of Walter Ruppel, Assoc. M. Am. Soc. C. E., to adapt them to the case of a tall, free-standing chimney, of truncated form, considered to act integrally with its foundation, the latter usually being a massive block of concrete. These modified formulas represent the so-called "hammer theory."[3] The results of the investigation of the Japanese chimneys were neither consistent nor reasonable and the writer came to the conclusion that the action of these chimneys had not been similar to that of a free, prismatic rod subjected to a single force.

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[32] "Experimentally Determined Dynamic Shears in a Sixteen-Story Model," by Lydik S. Jacobsen, and Robert S. Ayre, Jun. Am. Soc. C. E., *Bulletin*, Seismological Soc. of America, October, 1938.

[33] See *Civil Engineering*, March, 1932, p. 204.

[27] "The Stresses in a Free Prismatic Rod Under a Single Force Normal to Its Axis," by J. N. Le Conte, *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 968.

It may be questioned whether there is any earthquake phenomenon corresponding to a sudden blow or an impulsive force. It has been ascribed by some to the first longitudinal waves of the earthquake as distinguished from the large transverse waves which result in the violent swinging of structures. Seismographic records, however, have not generally indicated any such violent longitudinal waves. Possibly the sudden transition from the preliminary vibration of the earthquake wave to the first violent swing of its major portion might produce, in effect, a sudden blow.

A few years ago the writer's office had occasion to investigate the earthquake resistance of a free-standing, reinforced concrete chimney, 222 ft high. The theoretical resisting moment of the chimney was computed assuming that it would fail when the stress in the reinforcing steel reached 30 000 lb per sq in. In like manner, the shearing resistance of the chimney was computed assuming that the concrete would crack when it was stressed beyond 50 lb per sq in. in shear. Moments in the chimney were computed by the formulas of Le Conte and of Hoskins, using an assumed acceleration of 0.29 and ratios of the period of the earthquake to the natural period of the chimney of 1.25 and 1.50. The curves representing the results of these computations are shown in Figs. 9 and 10. Referring to Fig. 9, the moment curves, it will be seen that the zone in

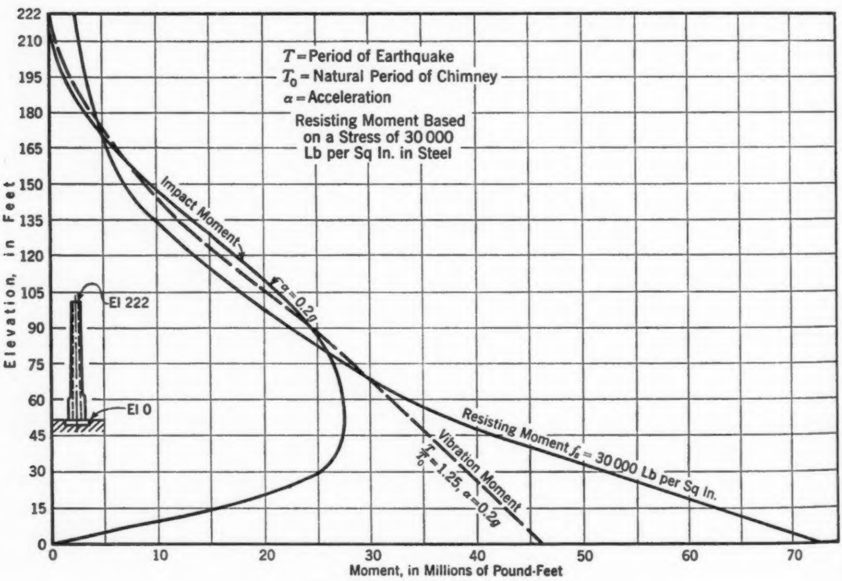

Fig. 9.—Moment Curves

which the active moments exceed the resisting moment is essentially the same for both the impact moment and the vibration moment. It is true that the excess amount of earthquake moment over the resisting moment is greater for the impact moment curve than for the vibration moment curve, although a not greatly different assumption of earthquake period and acceleration would

have produced a larger earthquake moment. For example, a ratio of periods $\frac{T}{T_0}$ of 1.00 instead of 1.25 would throw the vibration moment curve farther beyond the curve of resisting moment. It is also to be observed from Fig. 10,

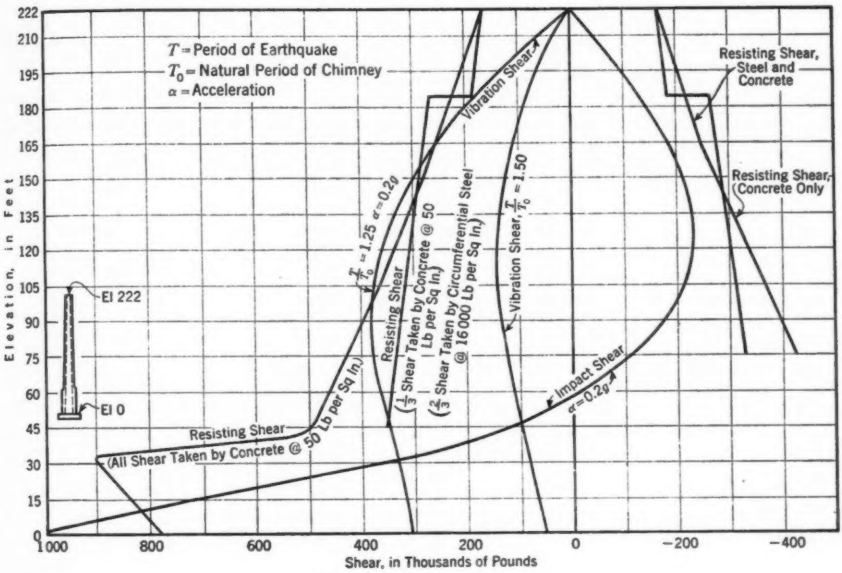

FIG. 10.—SHEAR CURVES

showing the shear curves, that the theoretical zone of over stress is essentially the same as that determined by the moment curves, although the over stress of shear is very much less than that of moment.

For many years engineers generally accepted the stated conclusion of the Japanese seismologist, the late F. Omori, that free-standing chimneys usually broke at two-thirds their height, and that their action during an earthquake somehow brought into play their centers of percussion, although the center of percussion of a chimney of the usual truncated form is nearer one-half its height than two-thirds. Indeed, the term "center of percussion" came to be associated with tall, free-standing chimneys and it was even thought by some that the value of acceleration should be doubled when used in the design of chimneys to provide for the effect of impulse.

In 1929 at the World Engineering Congress, I. Hiroi,[34] late Superintending Engineer of the Home Department of Tokyo, presented a paper in which he stated,

> "Of over 240 chimneys taller than 50 ft * * * more than 110 were totally wrecked, and over 40 seriously damaged. It is remarkable that the point at which the rupture took place was almost anywhere in the whole height, contrary to the old idea that it should be at two-thirds the

[34] "Prevention of Damage to Engineering Structures Caused by Great Earthquakes," by I. Hiroi, Paper No. 117, *Proceedings*, World Eng. Cong., Tokyo, 1920, Vol. 9, p. 25.

height in tall chimneys. The determination of the dimensions of a chimney sufficient to resist the actions of great earthquakes is not simple; but when proportioned by supposing the acceleration at the lower end to be equal to that of the ground, (say 3 000 m.m. per second per second), and assuming it to increase by 50% nearer to the top, it would be strong enough in most cases."

Concluding this discussion of chimneys, the writer believes it more probable that the earthquake fracture of free-standing chimneys is due to the effects of vibration rather than of impact, and results from failure in moment or shear, or a combination of both. It must be recognized that tall, free-standing chimneys, both of brick and of reinforced concrete, are special types of structures that are inherently vulnerable to earthquakes.

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DISCUSSIONS

PROPOSED IMPROVEMENTS FOR LAND SURVEYS AND TITLE TRANSFERS

Discussion

By George D. Whitmore, M. Am. Soc. C. E.

George D. Whitmore,[5] M. Am. Soc. C. E. (by letter).[5a]—The author has presented the case for better land surveys and title transfers quite clearly and convincingly. The writer, a practicing land surveyor, whose area of activity extends into several States, finds little to question or dispute in Professor Kissam's summary of the situation. Changes in land-surveying and title-proving procedures must come, and are in fact already slowly beginning, along lines suggested by the author.

There seems to be no question but that surveying equipment, skill, and techniques have been developed which, if applied, would largely eliminate land-boundary difficulties. The real problem seems to be that of bringing the equipment, skill, and techniques into general use. The Joint Committee of which the author is a member may find that there is relatively little difference of opinion as to what should be done; but they may find a discouraging amount of inertia in trying to get the recommended procedures actually used. It would seem that the Committee should organize itself for a long-range program of two parts: (1) Relatively simple (and apparently well advanced) to get agreement on sound basic procedures between the legal and engineering professions; and (2) (and much more difficult) to get these procedures adopted and into general practice.

There is one matter which the author's summary does not seem to cover, and which seems to the writer to be of greatest importance in the matter of improved land surveys—that is, improvement in the accuracy of the surveys and resulting descriptions or plats. Accurate co-ordinates will permit the accurate replacement of any lost corner by means of routine surveys; but they are a result of, and not a cause of, accurate surveys. Hence, it seems to the writer that more emphasis needs to be placed on accuracy of the surveys,

Note.—This paper by Philip Kissam was published in April, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[5] Chf. of Surveys, Maps and Surveys Div., TVA, Chattanooga, Tenn.

[5a] Received by the Secretary June 13, 1939.

whether these be control surveys, boundary traverses, or traverses connecting control surveys to the boundary surveys. It is the writer's belief that 80% to 90% of the trouble which surveyors now have in trying to re-establish lost or obliterated property corners would be eliminated if the original boundary surveys and deed descriptions had been accurate and free of blunders. If, also, all the accurate surveys had been tied together and computed on a State-wide co-ordinate system, there would be practically no difficulty of any kind in re-establishing lost corners. In other words, the simple requirement that boundary surveys shall be co-ordinated through traverse connections to co-ordinated control points, although of the greatest importance, is not in itself sufficient—there must be the requirement that these surveys shall be of a certain degree of accuracy and blunder-free.

The outstanding proof of this contention is the difficulty, almost everywhere encountered, of re-establishing in their original locations the township, range, section, and quarter-section corners of the older public-land surveys of the United States. These old subdivision surveys practically constitute in themselves a series of co-ordinate systems. Had the original surveys all been made with the same degree of accuracy, there would be relatively little difficulty in re-establishing the original locations of lost corners from the bearings and distances alone. The principal difficulty is that the old surveys, many of them handled by incapable contractors without proper supervision or inspection, were not of uniform accuracy, and also that they had more than their share of blunders. It is obvious that, co-ordinated or not, accurate original land subdivision surveys would eliminate most of the modern surveyor's difficulties in re-establishing lost section corners.

Another important point, from a public relations standpoint, is that surveyors should be careful to see that all published measurements and co-ordinates of the same course or point are in agreement. This has been stressed by A. H. Holt, M. Am. Soc. C. E.[6] To illustrate by a simple case, the writer saw considerable ridicule heaped on two reputable surveyors several years ago because they showed different distances for the same course on two adjoining record plats. The course in question was about 900 ft long; it was flat, open, and in every way conducive to accurate taping, and was the dividing line between two new building-lot subdivisions in a large Ohio city. Later investigation revealed that Surveyor A, whose tract was subdivided first, measured this course on a hot summer day, with his tape supported flat on the ground, and stretched under high tension. Surveyor B, whose tract was subdivided the following winter, measured the same course, at a temperature just a little above zero, with the tape suspended 2 or 3 ft above the ground, and stretched under relatively low tension. The two distances thus determined by the two surveyors differed by nearly 1 ft; yet both did accurate field taping. If the original field-computed distances had been corrected in the office for temperature, pull, and sag, the two final distances as determined by A and B would have agreed within a few hundredths of a foot. Publishing of the two uncorrected, different distances caused considerable confusion among, and ridicule from, public officials and others who had to use the two plats.

[6] "Trends in Boundary Surveying," by A. H. Holt, *Civil Engineering*, June, 1939, p. 359.

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DISCUSSIONS

THEORY OF LIMIT DESIGN

Discussion

BY MESSRS. L. H. NISHKIAN, AND F. G. ERIC PETERSON

L. H. NISHKIAN,[50] M. AM. SOC. C. E. (by letter).[50a]—Many experienced structural engineers have long recognized that, generally, in a redundant structure those structural elements best able to resist any given set of forces will ultimately carry the major part of the load if the structure is sufficiently loaded. (The term "redundant structure" as used herein does not necessarily mean a structure with more than the minimum number of members. A continuous beam or other indeterminate structure where all members or all parts of the same member are not working at equal efficiencies would be redundant within the present meaning.) In other words, within the elastic limit, load distribution is based on the principle of relative rigidities, whereas, beyond the elastic limit, the distribution would tend to approach a basis of relative strengths. Although the general principle is recognized by some and occasionally is used by a few, this idea has not been presented in American engineering literature, at least not quantitatively. Very ably, the paper deals with this subject and its logical corollaries.

In January, 1939, the writer observed an accidental condition in a coffer-dam which, under the usual assumptions of design, should have failed but did not. Fig. 38 shows this coffer-dam as designed and built. Fig. 38(b) shows the loads on the wales resulting from the water pressure when pumped out to a level just below the lower wale.

During the driving of the sheet-piling, apparently the corner of Wale A, Fig. 38, was caught by the lower end of the pile with the result that one end was pushed down, twisted, and entirely disengaged from Wale B. In other words, the sheet-piling caught at Point a, Fig. 39; the lower Wale A bent down; and the connection at Point c was broken loose, removing all support at the end

NOTE.—This paper by J. A. Van den Broek, M. Am. Soc. C. E., was published in February, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1939, by Messrs. John H. Meursinge, I. K. Silverman, Edward Godfrey, Basil Sourochnikoff, E. Mirabelli, C. M. Goodrich, George Winter, and Francis E. Simpson; and June, 1939, by Messrs. Joseph A. Wise, Alfred M. Freudenthal, Hans H. Bleich, Alfred S. Niles, and A. Floris.

[50] Cons. Engr., San Francisco, Calif.

[50a] Received by the Secretary May 23, 1939.

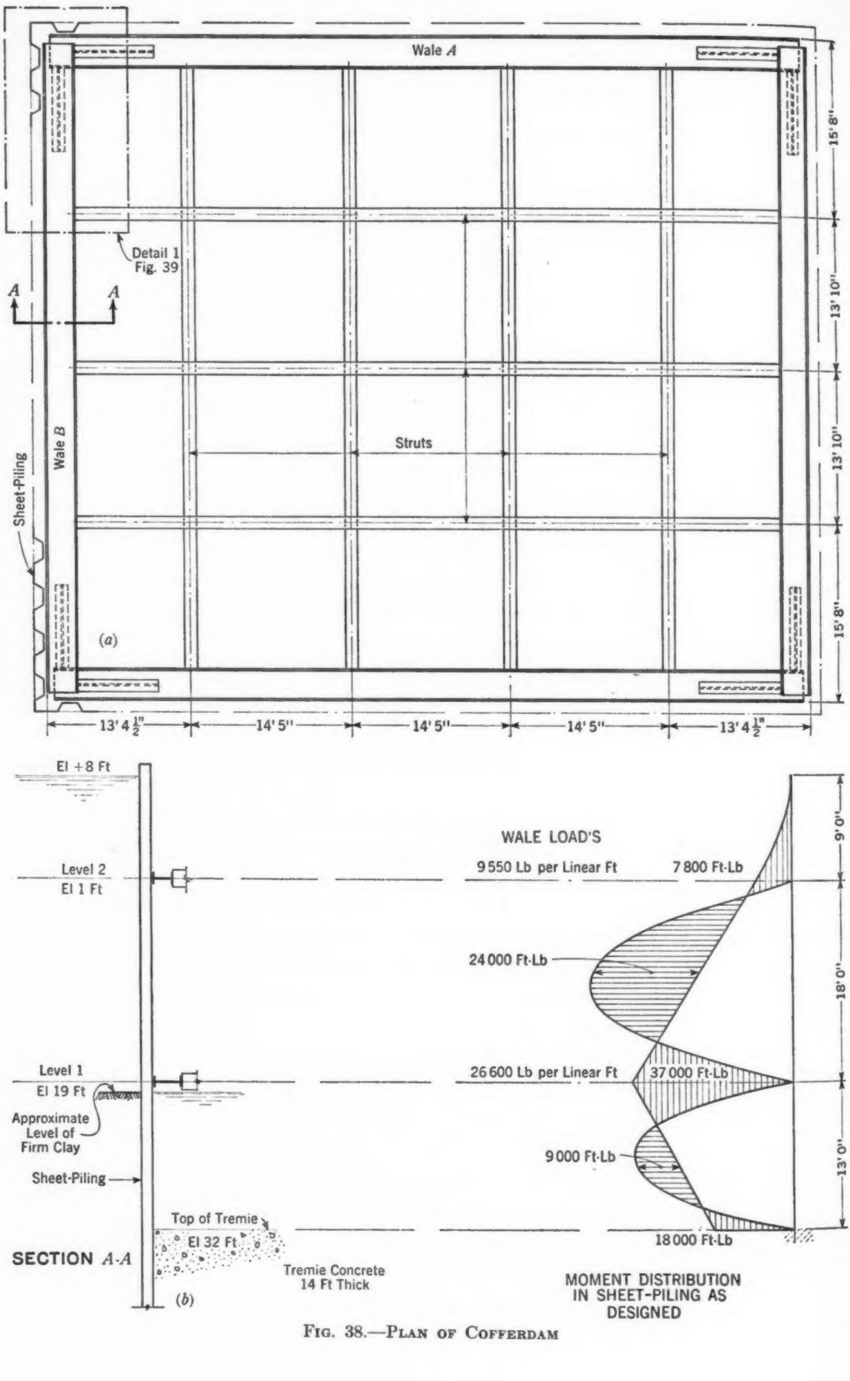

FIG. 38.—PLAN OF COFFERDAM

of Wale B. Wale A was so connected to Wale B that it acted as a column to take the end reaction of Wale B—about 185 kips. This accident was not noticed until the water was pumped down to the level below the lower wales—

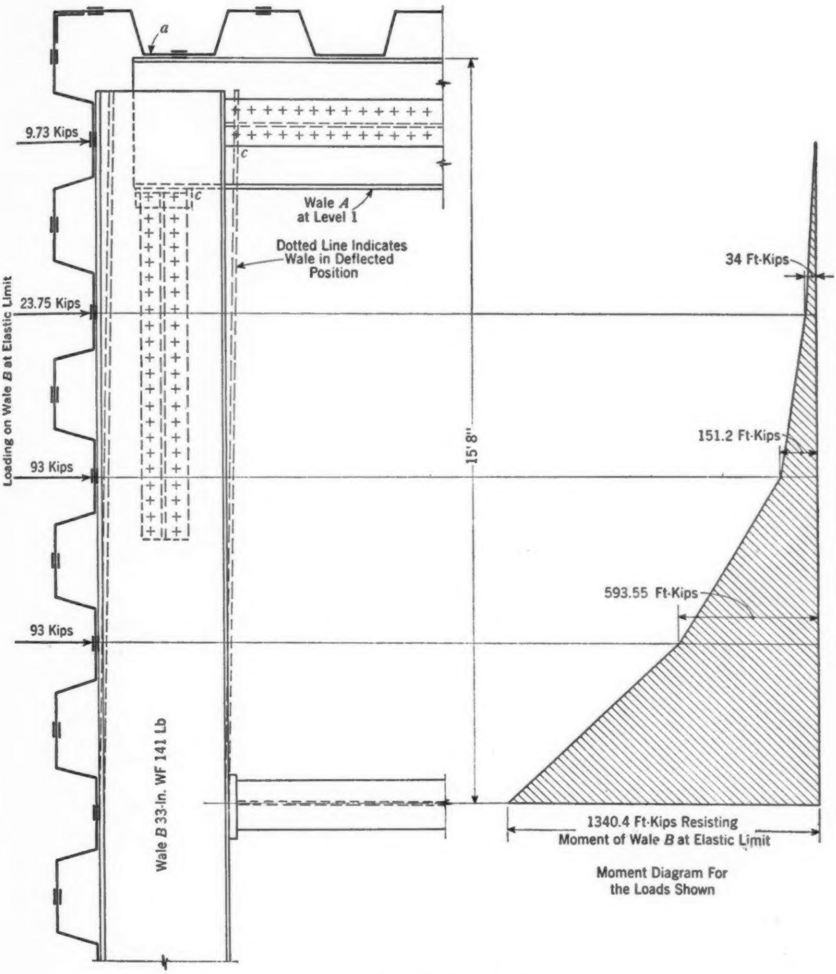

FIG. 39.—DETAIL 1, FIG. 38

about Elevation − 20 ft. With the water at this level, the load on the wales at Level 1 was more than 95% of the maximum load when fully pumped out. With Wale A pushed out of the way, Wale B had no end support and acted as a cantilever from the next strut—about 15.5 ft from the end.

Checking the capability of Wale B to carry this load as a cantilever, it was found to be entirely inadequate, even when using stresses of 36 kips per sq in. The capacity of the sheet-piling then was checked, assuming it to span from the

top wale to the tremie concrete and assuming the lower end fixed. This also was inadequate at 36 kips per sq in. Adding the maximum carrying capacity of the sheet-pile and Wale *B*, acting as a cantilever, both being stressed to the elastic limit, it was found that the resulting combined carrying capacity was still less than the water pressure acting on the coffer-dam. By all the ordinary rules, Wale *B* and the sheet-piling should have collapsed. Measurements at the

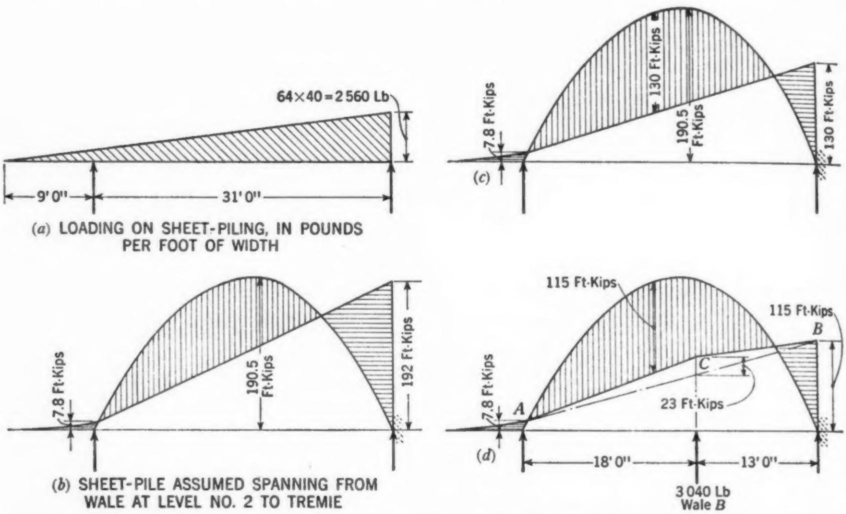

FIG. 40.—LOADING AND MOMENT DIAGRAMS (BACK PRESSURE OF WATER IN COFFERDAM NEGLECTED)

end of Wale *B* showed a horizontal deflection in excess of the elastic deflection of either the sheet-piling or the cantilever, Wale *B*, each stressed to the elastic limit of the steel. This corroborated the previous computation that the steel would be stressed beyond its elastic limit, but did not explain why it did not fail.

An analysis was then made along lines similar to those suggested by the author. If the first two sheet-piles continued to yield until the negative moment (never more than the resisting moment of the section at the elastic limit) at the tremie support became equal to the positive moment in the sheet-piling, and the remaining load was taken by Wale *B* as a cantilever, it was found that a balance would be struck although both the sheet-piling and the wale would be strained beyond their elastic limits. Figs. 39 and 40 show the moments resulting under this assumption. Fig. 40(*b*) demonstrates that a resisting moment of 192 ft-kips, within the elastic limit, is necessary to resist the loading given in Fig. 40(*a*). Even if a greater efficiency is effected by exceeding the elastic limit (see Fig. 40(*c*)), it is still necessary that the sheet-piling have a resisting moment of 130 ft-kips at the elastic limit to avoid failure. Since the resisting moment at the elastic limit is only 115 ft-kips, a reaction of 3 040 lb (as indicated in Fig. 40(*d*)) must be developed at Wale *B*, Fig. 38, to prevent deflection to the point of failure. Line *AB*, Fig. 40(*d*), was determined by the

resisting moment at the elastic limit of the sheet-piling (115 ft-kips). Line AC was drawn in such a manner that the moment intercepted at the center was also 115 ft-kips. This can be produced only by a reaction of 3 040 lb at Wale B.

In Fig. 39 the third sheet-pile from the end is shown reacting 93 kips on Wale B. Actually the elastic deflection of Wale B at this point will reduce this 93 kips and increase the 23 750-lb reaction at the second sheet-pile. An examination of the coffer-dam showed that the first three sheet-piles had been noticeably deflected, whereas the fourth was quite straight.

The author has presented a worth-while paper. A careful study of the ideas presented will lead to a more realistic understanding of the actual behavior of structures.

F. G. Eric Peterson,[51] Esq. (by letter).[51a]—Inasmuch as it focuses attention on possible revisions of design theory now used in engineering practice, this paper is a valuable one. Considerable study and much experimental work is necessary before the "Theory of Limit Design" can be accepted in actual design. The writer appreciates the new field of research opened by Professor Van den Broek's treatise.

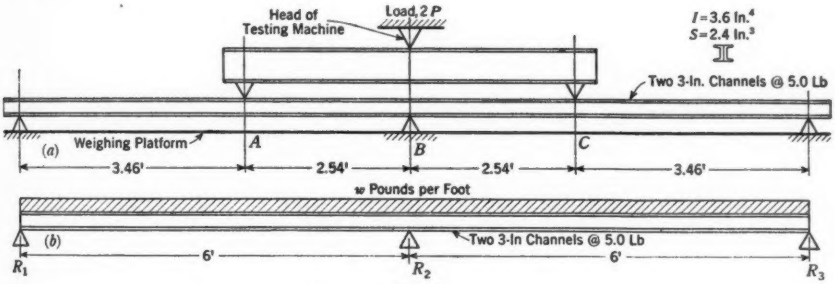

Fig. 41

The following discussion is confined to the "theory" as applied to beams and gives the results of tests applied to a beam resting on three supports. A test was made by the writer to determine the actual behavior of a continuous beam on three supports when subjected to the action of two concentrated loads as shown in Fig. 41(a). The position of the concentrated loads was such that the moment over the center support, as determined by the principle of continuity applied to an elastic structure, was a maximum. Before cutting the beam to the desired length a short piece was sawed out and submitted to the usual tensile test, the results of which were 38 600 lb per sq in. for the yield point and 61 700 lb per sq in. for the ultimate strength.

Referring to Fig. 42: For stresses below the elastic limit the moment over the center support and the moment under the concentrated loads will equal, respectively:

$$M_1 = -0.192 \frac{P L}{2} \qquad (27a)$$

[51] Instructor in Mathematics and Mechanics, Univ. of Minnesota, Minneapolis, Minn.

[51a] Received by the Secretary June 30, 1939.

and

$$M_2 = 0.133 \frac{P L}{2} \qquad \ldots\ldots (27b)$$

From the foregoing yield point stress, and the structural properties of the beam, it is found that the maximum resisting moment of the beam with no stresses exceeding the yield point is equal to 7 720 ft-lb, corresponding to a total load of 13 400 lb, or 6 700 lb on each span.

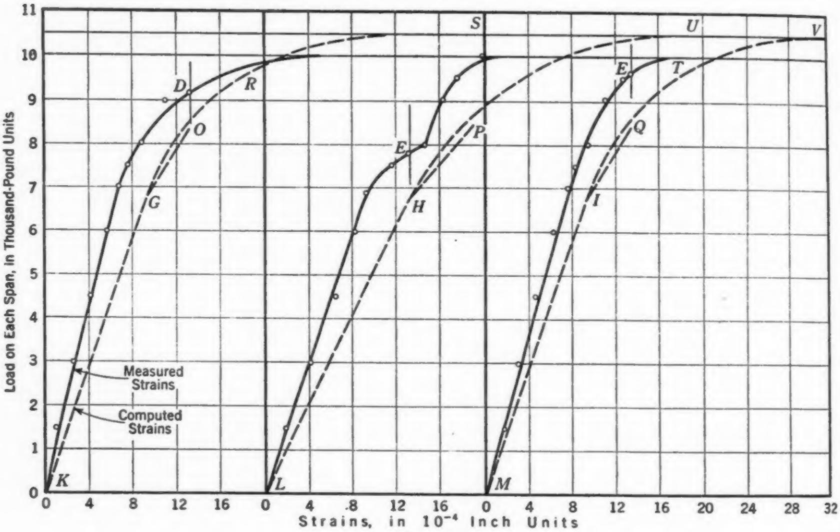

FIG. 42

According to Professor Van den Broek's theory the moment over the center support would reach a certain maximum and then remain constant at this value with increasing load until the moment under the loads reached this same value, at which time the beam would fail. From the test made by the writer this does not seem to be the case. Professor Van den Broek acknowledged the fact that the negative moments at the supports of his fixed beam increased somewhat as the load is increased. The writer believes from his test that as the load is increased the positive moment under the load and the negative moment at the supports increases appreciably until the failure load is reached. Of course, after the extreme fiber at a section has reached the yield point the resisting moment at this section increases more slowly but will not reach a constant value until failure defined as continued deflection without increase in load occurs. At this point the critical sections act as elastic hinges.

A separate test was made on a simple beam 5 ft long of the same cross-section as the beam continuous over three supports. When a single concentrated load at mid-span reached a value of 7 800 lb the beam continued to deflect without appreciable increase in load. This gives a resisting moment of $M = \frac{P L}{4} = 9\ 750$ ft-lb and would seem to indicate a stress of $s = \frac{9\ 750 \times 12}{2.4}$

= 48 750 lb. Of course, this is not a true stress since the outer fibers were stressed beyond the yield point somewhat before failure occurred; but it is comparable to the usual modulus of rupture. Almost all of the cross-section is at the yield point stress but has not begun to approach the ultimate strength.

Assuming that the moment over the center support and also under the loads could reach this value of 9 750 ft-lb before deflection without increase in load, the load on each span could be 10 500 lb or the total load 21 000 lb. This value is in close agreement with the test made as the maximum load was found to be somewhat more than 20 000 lb.

The beam was loaded as indicated in Fig. 41(*a*). At each load increment strain gage readings were taken, four at each of the three sections *A*, *B*, and *C*, using a 2-in. gage. The graphs of Fig. 42 show the strains at the extreme fibers corresponding to the loads. The solid lines show the measured strains; and they appear to be somewhat smaller than those computed from the loads indicated by the testing machine, probably attributable to the fact that the strain gage records average strains over a distance which has a varying bending moment. Furthermore, the gage points were placed at the edge of the flanges and therefore a little closer to the neutral axis than the extreme fibers. The vertical lines at *D*, *E*, and *F*, Fig. 42, indicate the yield point strain (approximately). The line *L H* is the computed load strain line at Section *B* up to a oad of 6 700 lb. At Point *H* the extreme fibers at Section *B* have reached the yield point. Lines *K G* and *M I* represent the load strain line for Sections *A* and *C*, respectively. If the moment at Section *B* remained constant when the extreme fiber reached the yield point the load strain curve at Points *A* and *C*, Fig. 41(*a*), would be as *G D* and *I Q*, respectively, up to the yield point strain of the extreme fibers at these sections. However, since Section *B* gives an increasing resisting moment beyond this point the strains at Sections *A* and *C* would increase less rapidly and would follow curves similar to *G R S* and *I T V* instead. Points *S* and *V*, Fig. 42, are on a horizontal line at the theoretical maximum load and are obtained by neglecting a section 0.5 in. above and below the neutral axis. This section has a moment of inertia about one one-hundredth of the total section.

Applying a factor of safety of 2, the maximum total load would be 10 500 lb (calculated) or, the load *P* on each span would be 5 250 lb. Note that the load necessary to cause a yield-point fiber stress at the center support is 13 400 lb or about one-third higher. Allowing a maximum fiber stress of 18 000 lb per sq in. (which is common practice) the allowable total load would be $P = \frac{2\,M_1}{0.192\,L}$ = 6 250 lb, or 3 125 lb on each span. This load allows a factor of safety of about 2 for a yield point stress of 38 600 lb but a factor of safety of 3.68 for the structure as a whole. It would thus seem that for statically indeterminate structures that the theory of limit design shows that higher loads can be used with safety.

The Case of a Uniform Load Over Three Supports.—The ordinary theory and the theory of limit design is applied herein to a uniformly distributed load using the same beam and the same span lengths as in the preceding example (see Fig. 41(*b*)). In the ordinary theory of design, assuming a maximum stress of

18 000 lb per sq in.: $M_1 = M_3 = 0$; $4\,M_2 = -\frac{w\,l^2}{2}$; $M_2 = -\frac{w\,l^2}{8}$; and, $w = \frac{8\,M_2}{l^2} = \frac{8 \times 18\,000 \times 2.4}{36 \times 12} = 800$ lb per ft, safe load.

In the theory of limit design, the section was found to resist 9 750 ft-lb of moment before appreciable deflection occurred. Assuming that M_2 reaches this value, the minimum load per foot necessary to cause the same positive moment at some point in the beam, is obtained as follows: $R_1 = \frac{\frac{1}{2}\,w\,l^2 - 9\,750}{l} = 3\,w - 1\,625$. The moment at any point in the beam is

$$M = R_1\,x - \frac{w\,x^2}{2} = (3\,w - 1\,625)\,x - \frac{w\,x^2}{2} \qquad \text{(27c)}$$

Setting Equation (27c) equal to 9 750 ft-lb, $w = \frac{19\,500 + 3\,250\,x}{6\,x - x^2}$; $\frac{dw}{dx} = \frac{3\,250\,(6\,x - x^2) - (19\,500 + 3\,250\,x)\,(6 - 2\,x)}{(6\,x - x^2)^2} = 0$; and, $x = 6\,(\sqrt{2} - 1)$ gives a minimum value of $w = 3\,160$ lb. The value of $x = l\,(\sqrt{2} - 1)$ found herein holds for any maximum resisting moment of any beam of two spans of equal length on three supports.

Applying a factor of safety of 2 to this load results in a safe load of 1 580 lb per ft. The maximum deflection with this load is approximately 0.12 in. Since this load is almost sufficient to cause yield-point stress over the center support the practicing engineer might wish to reduce it. However, even if it is reduced to 1 200 lb it is still 50% greater than the value obtained by the customary method. Note that, in this method, the concept of stress did not enter.

The writer wishes to acknowledge the suggestions and assistance of Paul Andersen, Assoc. M. Am. Soc. C. E., in the preparation of this discussion.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

WIND BRACING IN STEEL BUILDINGS

SIXTH PROGRESS REPORT OF SUB-COMMITTEE NO. 31 COMMITTEE ON STEEL OF THE STRUCTURAL DIVISION

Discussion

By Messrs. Arthur G. Hayden, Robins Fleming, and C. M. Goodrich

Arthur G. Hayden,[29] M. Am. Soc. C. E. (by letter).[29a]—Present knowledge of the correlation between wind velocities and wind pressures on large areas, such as the face of a tall building, is inadequate. However, it is not proper to conclude that, as a consequence, the refinements of mathematical analysis suggested in the Report are of little more than academic interest and that further study along such lines would be of little practical value.

Older designing engineers in the profession can readily recall the period when building frames were designed with no regard for the effects of continuity or column shortening or other refinements such as were under discussion. The advance of building construction compelled a revision of design processes because instances of disconcerting defects multiplied. Although they did not result in complete failure, these defects sometimes resulted in loss of tenants and entailed expensive repairs. Tall building construction of to-day was made possible only by a critical analysis of out-moded design processes and subsequent refinements in mathematical methods. The question now is whether design processes have reached a stage that would warrant a discontinuation of mathematical investigations, such as are presented in the Report, until investigations relating to wind pressures have "caught up," as some engineers seem to think.

Even when the latter field has been thoroughly explored and complete data on wind pressures are at hand, the element of judgment will not be eliminated. The rules for intensity of wind pressure to be used in design in connection

Note.—The Sixth Progress Report of Sub-Committee No. 31, Committee on Steel of the Structural Division, was presented at the meeting of the Structural Division, New York, N. Y., January 19, 1939, and was published in June, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the report.

[29] Cons. Civ. Engr., New York, N. Y.

[29a] Received by the Secretary August 2, 1939.

with assumed working stresses will still be arbitrary. Nevertheless, the value of continued studies of processes of mathematical analysis will not be diminished. The more elements of doubt that are removed—so much the closer can designers approach the truth.

The work being done by the Committee is clarifying the issue as to which elements of design are important and which can safely be neglected in particular cases. The result will be a safe and sane simplification of some of the current processes. A further advantage will be an increased knowledge of the structural action of tier buildings so that designers will develop that intuition without which mathematical processes are like tools in clumsy hands. When more is learned about the correlation between wind velocities and wind pressures, about wind gusts and vibration and about allowances that can be made for the walls within the frame, the results of the present field of investigation will permit the designer to take better advantage of the new knowledge. Uniform advance along the entire front would be an advantage but structural designers have not yet reached such an ideal stage of planned economy.

Robins Fleming,[30] Esq. (by letter).[30a]—Those interested in wind action on buildings will find this Report valuable. Each of its four divisions treats a phase of the subject in a masterly manner. Emphasis is laid on finding wind reactions in a framed structure before attempting to find wind stresses. A just criticism of conventional methods is that the wind is assumed to follow a predetermined path, regardless of the stiffness of members through which it must pass. A method of obtaining "K-percentages" is given $\left(K = \frac{I}{l} = \text{stiffness of member}\right)$. The method of stress analysis based on these "K" values could well have been prefaced by a brief outline of other methods.

In presenting the slope-deflection method mentioned in the Report,[2] Professors Wilson and Maney reject the cantilever and portal methods (the so-called Fleming methods) presented[31] by the writer in 1913. Subsequently, this material was rewritten in better form.[32] In 1912 the writer used the portal method in the design of an eighteen-story building in Atlanta, Ga. One year later he used the cantilever method in designing a twenty-two-story building in Philadelphia. The slope-deflection method is impracticable in the actual design of tall buildings.

The method of K-percentages (heading: "Determination of Column Wind Reactions from Member Stiffnesses: Basis of Witmer Method of K-Percentages") is practicable and has certain advantages over many other methods that have been advanced in the past ten or fifteen years. Like the others, it requires a preliminary design. The query is raised: Why not make a pre-

[30] Structural Engr. (*Retired*), New York, N. Y.

[30a] Received by the Secretary July 15, 1939.

[2] *Bulletin No. 80*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill.

[31] "Wind Bracing Without Diagonals for Steel-Frame Buildings," by Robins Fleming, *Engineering News*, Vol. 64, March 13, 1913, p. 492.

[32] "Wind Stresses in Many-Storied Buildings," by Robins Fleming, *Engineering*, Vol. CXXV, May 25, 1928, p. 625. See also "Wind Stresses in Buildings," by Robins Fleming, John Wiley & Sons, Inc., New York, N. Y. (1930).

liminary design by one of the conventional methods, say, the portal method, and revise it in accordance with the stiffness of members, especially of columns? The "Witmer" method, it may be called.

The percentage in variation of wind pressure to be assumed, or of unit stresses to be used, as given in different building codes is as great as, if not greater than, the "error" in determining wind stresses. In the building codes of cities the permissible loads to be carried by the floors of commercial buildings vary as much as 50%, and occasionally more. Again, masonry walls between columns, concrete floors and tile partitions, offer a greater resistance to wind action than the steel frame itself. To quote D. A. Molitor, M. Am. Soc. C. E.: "In the face of these facts, what useful purpose can be served by accurate methods of stress analysis?"[33] It is interesting to note from the Report that the effect of column deformation as calculated in a twenty-six-story bent with a height-to-base ratio of 5 to 1 was scarcely appreciable; and, that the performance of the bent was substantially "portal."

In their consideration of the torsional effects of wind on buildings, the Committee touch on a phase about which little has been written. After the Florida hurricane of 1926 it was observed that an entire structure twisted when one end was stiffer than the other.[34] The same tendency is present when the pressure on a face is not uniformly distributed. The center of pressure in both cases is eccentric with the center of resistance. In most buildings the subject has only academic value. For tall buildings of rectangular shape, where the length is much greater than the width, it is well to consider possible torsion.

The fourth division of the Report, "Magnitude of the Assumed Wind Force on Tall Buildings," is its most important part. Messrs. Hugh L. Dryden and G. C. Hill[35] state that they see no hope of advance of knowledge unless wind loads are investigated rather than wind stresses. No common agreement has been reached as to exactly what wind pressure should be assumed in designing a tall building. In view of the personnel of the Committee and the wide range of suggestions they considered, their recommendations are entitled to weight. However, the writer wishes that, for the upper 200 ft of buildings 500 ft high, they had recommended more than 20 lb per sq ft pressure. His preference would be:

(1) For the first 300 ft above the ground, a uniformly distributed force of 20 lb per sq ft; and,

(2) For that part of a building above the 300 ft level a force of 20 lb per sq ft plus 2.5 lb per sq ft for each additional 100 ft in height.

Recommendation (3) he would omit.

It is doubtful if, in the future, commercial buildings will be built more than 500 ft in height. In 1939 there were said to be 36 buildings in New York

[33] "Structural Engineering Problems," by D. A. Molitor, Chapter 7.

[34] "Final Report of the Committee of the Structural Division on Florida Hurricane," *Proceedings*, Am. Soc. C. E., August, 1928, p. 1757.

[35] *Scientific Paper No. 523*, April 3, 1926, Bureau of Standards, Vol. XX, pp. 697–732.

City, and 19 in other cities, 500 ft or more high.[36] Three years previously[37] the report was 35 and 19, respectively.

The last word about wind bracing has not been said. As stated in the Report: "A study of service and maintenance records of buildings of the type discussed would be of incalculable value in correlating theory with actual performance." Alfred C. Bossom, after a practice of about twenty years as an architect in the United States, returned to England, entered public life and became a member of Parliament. In his book, "Building to the Skies" (1934), he states: "The science of wind-bracing has hardly begun." According to aerodynamics there is a negative pressure on roofs, especially flat roofs, and on the leeward side of buildings. No mention of this is made in the conclusions of the Report.

C. M. Goodrich,[38] M. Am. Soc. C. E. (by letter).[38a]—It has been said that the engineer should do with one dollar what any fool could do with two. One might add that the engineer must often obtain his solutions in days, where the mathematician-physicist may take weeks or months. In the Panchatantra one may read:

"Scholarship is less than sense,
Therefore seek intelligence."

In the "Second Report of the Steel Structures Research Committee,"[39] as a result of field tests, "It was found that when an apparently symmetrical two-bay three-story frame was loaded the stresses at certain corresponding sections differed by as much as 55.5 per cent."

It is generally recognized that reasonably substantial buildings stand up, whether computed by one method or another.

The Sixth Progress Report of Sub-Committee No. 31 on Wind Bracing in Tier Buildings offers a method of great value to design offices, uniting a logical view-point, simplicity, and brevity, with ample accuracy for the purposes of design. It would seem that here sound scholarship was joined to good sense. It would seem appropriate to call this the "Witmer method."

George Unold[40] has given the most brief and orderly presentation of an "exact" method known to the writer. It is based on the "End Tangent Method," which is another name for Mohr's elastic weights or Greene's area moments. However, it is too long for practical purposes. The same theory is the basis of Adolf Kleinlogel's "Rahmenformeln," first published in 1914. A pamphlet by the late C. E. Greene, M. Am. Soc. C. E., entitled "Partially Braced Frames:—Portal Bracing," undated, but of which a then old copy was given the writer about 1905, develops this method of slopes and deflections for

[36] "The World Almanac and Book of Facts," The New York World-Telegram, New York, N. Y., 1939, p. 441.

[37] *Loc. cit.*, 1936, p. 513.

[38] Chf. Engr., The Canadian Bridge Co., Ltd., Walkerville, Ont., Canada.

[38a] Received by the Secretary July 26, 1939.

[39] "Second Report of the Steel Structures Research Committee," Dept. of Scientific and Industrial Research, Great Britain, p. 306; published in 1934, in London, by His Majesty's Stationery Office.

[40] "Die Praktische Berechnung der Stahlskelettrahmen," by George Unold, W. Ernst und Sohn, Berlin, 1933.

pin-based and fixed-based portals of eight varieties. In the United States the method appears to have been renamed Slope Deflection in 1915.

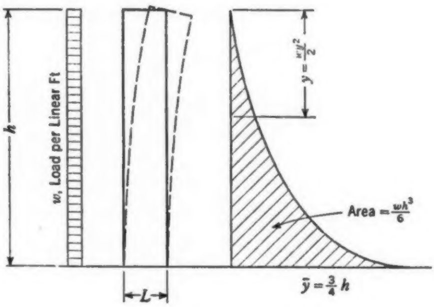

FIG. 13

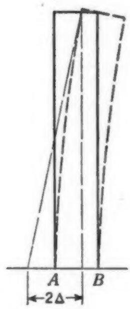

FIG. 14

Referring to Fig. 13, a simple derivation of Equation (7), of the Report,[16] follows: In a beam, if shear is ignored, by the Second Theorem of Area Moments

$$\Delta_c = \frac{w\,h^3}{6} \times \frac{3}{4}\,h \times \frac{1}{E\,I} = \frac{w\,h^4}{8\,E\,I} \quad \text{..............(10)}$$

Assuming a straight-line distribution of stress over the base

$$I = \frac{M\,y}{S} = \frac{w\,h^2}{2} \times \frac{0.5\,L}{S} = \frac{w\,h^2\,L}{4\,S} \quad \text{..............(11)}$$

Substituting the value of I obtained in Equation (11) in Equation (10):

$$\Delta_c = \frac{w\,h^4}{8\,E}\,\frac{4\,S}{w\,h^2\,L} = \frac{h^2\,s}{2\,E\,L} \quad \text{..............(12)}$$

If the moment of inertia is assumed uniformly increasing from top to base (since wind stress varies as $\frac{w\,y^2}{2}$, whereas dead loads and live loads vary as y) and, if I_B is the moment of inertia at the base:

$$\Delta_c = \int_0^h \frac{w\,y^3\,dy}{2\,E\,I_B\,\frac{y}{h}} = \frac{w\,h^4}{6\,E\,I_B} \quad \text{..............(13)}$$

and this would give a constant of $\frac{2}{3}$ instead of $\frac{1}{2}$.

In a block the value of I varies as $\frac{L^3}{12}$; in a two-bay bent it varies (if column loads vary as the floor areas they support) as $2\left[\frac{L}{4} \times \left(\frac{L}{2}\right)^2\right] = \frac{L^3}{8}$. In view of such possible differences it might be desirable to use Equation (13), with I_B in the denominator.

[16] *Bulletin No. 93*, Ohio State Univ. Eng. Experiment Station, Columbus, Ohio, p. 23.

It would seem possible to devise a simple formula to cover the case in which the sway of a building is to be calculated from the girder deflections alone. Assuming the building to bend in parabolic form, a tangent from the top will cut the ground level at $2\,\Delta$ from the base of the column. The angle at the top, by the First Theorem of Elastic Weights, is

$$\alpha = \frac{M_{AB}\frac{L}{3} - M_{BA}\frac{L}{6}}{E\,I} \quad \text{(14)}$$

This would seem to accord well with the methods of the Sixth Report. In using such a short cut the average values, as used in the Report, should be the basis of calculations, as angles at the top might well be somewhat out of accord with the angles from the bottom to a point a few stories down from the top.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

DESIGN OF AN OPEN-CHANNEL CONTROL SECTION

Discussion

BY MESSRS. V. L. STREETER, EMERY H. WILLES, AND ROBERT O. THOMAS

V. L. STREETER,[4] ASSOC. M. AM. SOC. C. E. (by letter).[4a]—The theoretical solution for an open-channel control section, presented in this paper, will be of value to the designing engineer. The writer, however, found the derivation difficult to understand, particularly as to how Equation (2) requires the condition of critical flow, and why the denominator, instead of the numerator, of Equation (5) is set equal to zero for maximum discharge. Although the theoretical solution given by the author results in correct final equations, the two foregoing points so obscured the derivation that the writer attempted to find a more simple solution. For example, combining Equations (1),

$$\frac{Q^2}{2\,g\,A_c^2} = H_p - d_c \quad\text{.....................}(19)$$

As critical flow requires maximum discharge for a given available energy head, Equation (19) will be differentiated with respect to A_c, considering H_p a constant; thus

$$\frac{Q\,dQ}{g\,A_c^2\,dA_c} - \frac{Q^2}{g\,A_c^3} = -\frac{dd_c}{dA_c} \quad\text{.................}(20)$$

Setting $\frac{dQ}{dA_c}$ equal to zero for maximum Q, a critical flow relation results; thus:

$$\frac{dd_c}{dA_c} = \frac{Q^2}{g\,A_c^3} \quad\text{.......................}(21)$$

As $B_c\,dd_c = dA_c$, Equation (21) contains the two unknowns B_c and A_c. To determine another relationship involving the two unknowns, differentiate

NOTE.—This paper by Karl R. Kennison, M. Am. Soc. C. E., was published in May, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[4] Associate Engr., U. S. Section, International Boundary Comm., El Paso, Tex.

[4a] Received by the Secretary May 31, 1939.

Equation (19) with respect to Q, with H_p a variable. Then:

$$\frac{dH_p}{dQ} = \frac{Q}{g\,A_c^2} - \frac{Q^2}{g\,A_c^3}\frac{dA_c}{dQ} + \frac{dd_c}{dQ} \qquad (22)$$

Eliminating dd_c in Equations (21) and (22):

$$\frac{dH_p}{dQ} = \frac{Q}{g\,A_c^2} \qquad (23)$$

which contains one unknown, A_c. Equations (21) and (23) are equivalent to Equations (10) and (6), as $\frac{dH_p}{dQ} = m$. When the desired rating curve is given in the form:

$$d_p = f_1(Q) \qquad (24)$$

and

$$A_p = f_2(d_p) = f_2[f_1(Q)] \qquad (25)$$

(in which f_1 and f_2 are known functions) $\frac{dH_p}{dQ}$ may be determined from:

$$\frac{dH_p}{dQ} = \frac{Q}{g\,A_p^2} - \frac{Q^2}{g\,A_p^3}\frac{dA_p}{dQ} + \frac{dd_p}{dQ} \qquad (26)$$

for any value of Q. Equation (26) is Equation (22) with Subscript c replaced by Subscript p.

In designing the control, Equations (19), (21), (23), and (26) are needed. From Equation (26) $\frac{dH_p}{dQ}$ is determined for several values of Q throughout the range of capacity of the structure. With these values of $\frac{dH_p}{dQ}$, A_c is found from Equation (23), and with A_c known, values of $\frac{dd_c}{dA_c} = \frac{1}{B_c}$ are found from Equation (21). Corresponding values of d_c are obtained from Equation (19).

If friction is to be taken into account, it may be estimated for the length of travel between the piezometer section and control section, for various discharges, thus:

$$h_f = f_3(Q) \qquad (27)$$

Equation (23) then becomes:

$$\frac{dH_p}{dQ} - \frac{dh_f}{dQ} = \frac{Q}{g\,A_c^2} \qquad (28)$$

and Equation (19):

$$\frac{Q^2}{2\,g\,A_c^2} = H_p - h_f - d_c \qquad (29)$$

Equations (21) and (26) remain the same.

EMERY H. WILLES,[5] JUN. AM. SOC. C. E. (by letter).[5a]—The author needs to be commended on his unique approach to a problem, which, due to the

[5] Hydrographer, Utah Copper Co., Salt Lake City, Utah.

[5a] Received by the Secretary June 26, 1939.

mathematical complicity at first encountered, leads the pioneer investigator to despair of finding a simple theoretical solution. However, by expressing the width B_c and the depth d_c of the controlling section in terms of the slope m and intercept n of the tangent to the total energy-curve, followed, in turn, by expressing m and n in terms of the slope S of the rating curve and the known elements of the piezometer section, the author has presented a simple and effective solution to this seemingly non-solvable problem.

The writer, however, is unable to follow the author from Equation (5) to Equation (6). It is obvious from Equation (5) that $Q^2 - f(A_c) = 0$; also from Equation (6) that $Q - A_c^2\, g\, m = 0$. Then, by substitution, Equation (5) becomes the indefinite relationship $\frac{0}{0} = 0$. Equation (6) may also be obtained if $\frac{dQ}{dA_c} = \infty$ instead of 0. Thus, since $\frac{1}{\infty} = 0$, $\frac{A_c\,(Q - A_c^2\, g\, m)}{Q^2 - f(A_c)} = 0$; and, $Q = A_c^2\, g\, m$. This is very perplexing and led the writer to investigate $f(A_c)$ in Equation (5) as follows: Referring to Equation (4)

$$\frac{1}{2\,g}\left(\frac{2\,A_c^2\,Q\,dQ - 2\,Q^2\,A_c\,dA_c}{A_c^4}\right) = m\,dQ - dd_c = \frac{A_c\,Q\,dQ - Q^2\,dA_c}{g\,A_c^3} \quad\ldots\ldots(30)$$

Simplifying, and solving for $\frac{dQ}{dA_c}$, $\frac{dQ}{dA_c} = \frac{Q^2 - g\,A_c^3\,\frac{dd_c}{dA_c}}{A_c\,(Q - A_c^2\,g\,m)} = 0$; and,

$$Q = \pm\sqrt{g\,A_c^3\,\frac{dd_c}{dA_c}} \quad\ldots\ldots(31)$$

In any section $dA_c = B_c\,dd_c$; hence

$$\frac{dd_c}{dA_c} = \frac{1}{B_c} \quad\ldots\ldots(32)$$

and

$$Q = \sqrt{\frac{g\,A_c^3}{B_c}} \quad\ldots\ldots(33)$$

The foregoing investigation is leading the writer no nearer Equation (6) or the final solution. Therefore, since the author's solution revolves around Equation (6), the writer presents the following derivation: From Equations (1*c*) and (2)

$$Q = \frac{H_p}{m} - n = \frac{h_{vc} + d_c}{m} - n \quad\ldots\ldots(34)$$

and, because m and n are constant at the critical section, $dQ = \frac{dh_{vc} + dd_c}{m}$; and,

$$\frac{dQ}{dA_c} = \frac{1}{m}\left(\frac{dh_{vc}}{dA_c} + \frac{dd_c}{dA_c}\right) \quad\ldots\ldots(35)$$

From Equations (1*a*) and (1*b*): $h_{vc} = \frac{V_c^2}{2\,g} = \frac{Q^2}{2\,g\,A_c^2}$; and, $Q^2 = 2\,g\,h_{vc}\,A_c^2$.

Furthermore, $2\,Q\,dQ = 4\,g\,h_{vc}\,A_c\,dA_c + 2\,g\,A_c^2\,dh_{vc}$; and,

$$\frac{dQ}{dA_c} = \frac{g\,A_c\left(2\,h_{vc} + A_c\,\dfrac{dh_{vc}}{dA_c}\right)}{Q} \quad\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

Combining Equations (35) and (36) and solving for Q:

$$Q = \frac{m\,g\,A_c\,(2\,h_{vc}\,dA_c + A_c\,dh_{vc})}{dh_{vc} + dd_c} \quad\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

From Equations (1*a*), (1*b*), and (1*c*) $H_p = d_c + h_{vc} = d_c\dfrac{V_c^2}{2\,g} = d_c + \dfrac{Q^2}{2\,g\,A_c^2}$; $Q^2 = 2\,g\,(H_p\,A_c^2 - A_c^2\,d_c)$; and, since H_p is assumed constant, $2\,Q\,dQ = 2\,g\,(2\,H_p\,A_c\,dA_c - 2\,A_c\,d_c\,dA_c - A_c^2\,dd_c)$. Finally, at the critical section, $\dfrac{dQ}{dd_c} = \dfrac{g}{Q}\left(2\,H_p\,A_c\,\dfrac{dA_c}{dd_c} - 2\,A_c\,d_c\,\dfrac{dA_c}{dd_c} - A_c^2\right) = 0$; and, solving for H_p:

$$H_p = A_c\left(\frac{2\,d_c\,\dfrac{dA_c}{dd_c} + A_c}{2\,A_c\,\dfrac{dA_c}{dd_c}}\right) = d_c + \frac{A_c}{2}\,\frac{dd_c}{dA_c} \quad\ldots\ldots\ldots\ldots (38)$$

From Equation (32) $\dfrac{dd_c}{dA_c} = \dfrac{1}{B_c}$; and, $H_p = d_c + \dfrac{A_c}{2\,B_c}$. Combining with Equation (1*c*):

$$h_{vc} = \frac{A_c}{2\,B_c} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Writing the differential equation of Equation (39):

$$dh_{vc} = \frac{1}{2}\left(\frac{B_c\,dA_c - A_c\,dB_c}{B_c^2}\right) = \frac{1}{B_c}\left(\frac{dA_c}{2} - \frac{A_c}{2\,B_c}\,dB_c\right)$$

$$= \frac{1}{B_c}\left(\frac{dA_c}{2} - h_{vc}\,dB_c\right) \quad\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

Combining Equations (32), (37), (39), and (40), and solving algebraically for Q:

$$Q = \frac{m\,g\,A_c^2\left(\dfrac{3}{2}\,dA_c - h_{vc}\,dB_c\right)}{\dfrac{3}{2}\,dA_c - h_{vc}\,dB_c} = m\,g\,A_c^2 \quad\ldots\ldots\ldots\ldots (41)$$

ROBERT O. THOMAS,[6] JUN. AM. SOC. C. E. (by letter).[6a]—Considerable credit is due the admirable analysis made by the author. The writer is constrained, however, to offer a few criticisms and suggestions with a view to a more complete understanding of the hydraulic action of a control such as has been proposed. To that end the following equations are presented, using only

[6] Junior Engr., with Donald M. Baker, Cons. Engr., Los Angeles, Calif.

[6a] Received by the Secretary June 30, 1939.

one intermediary, m, to determine the elements of the control section:

$$d_c = H - \frac{m\,Q}{2} \qquad (42)$$

$$B_c = \frac{0.249\sqrt{H_p - d_c}}{Q\,m^2} \qquad (43)$$

$$m = \frac{S\,(A_p - 2\,B_p\,h_{vp})}{A} + \frac{V}{A\,g} \qquad (44)$$

or,

$$m = S + \frac{2\,h_{vp}}{Q}\,(1.00 - V_p\,B_p\,S) \qquad (45)$$

The use of Equations (42) to (44) will require less computation than those given in the paper.

The mathematical correctness of the method proposed by the author is beyond question, but the writer entertains doubts as to the practical feasibility in the use of such a control. Analysis of the flow through the control and in the channel above the control leads to the conclusion that the incorporation of such a design would seriously affect the conditions of flow in the canal. It is a condition of the design that the total energy head on the moving water prism shall be the same, neglecting friction, at the piezometer section and at the control. Since the flow through the control is at critical depth, this condition definitely establishes the water surface at both points. It is also a condition of the design that the canal shall carry less water than it otherwise would, if the control station were omitted. This condition alone leads to the conclusion that the channel could not be designed economically, because the cross-section would, of necessity, be larger than would ordinarily be required to convey the given flow.

However, the principal difficulties in the design arise in the section of channel which lies above the rating station. The elevation of the water surface at the piezometer section is not that which can be maintained in the channel above, due to the fact that the flows and depths at this section are not those which would result from uniform flow. Assuming a concrete-lined channel, with "n" equal to 0.014, Table 2 (Column (3)) gives the slopes necessary to maintain a uniform flow in the channel above the piezometer section.

TABLE 2.—CHANNEL SLOPE NECESSARY TO MAINTAIN UNIFORM FLOW

Depth, in feet	Flow, Q, in cubic feet per second	Channel slope
3.00	28.30	0.00043
2.50	23.21	0.00058
2.00	18.12	0.00061
1.60	14.04	0.00072
1.20	9.97	0.00087
0.80	5.89	0.00104
0.418	2.00	0.00090

TABLE 3.—FLOW IN A TRAPEZOIDAL CHANNEL (TEN-INCH SLIDE-RULE COMPUTATIONS)

Depth d_n, in feet	Velocity V_n, in feet per second	HEAD,* IN FEET		
		H_{vn}	H_n	H_p
2.28	3.51	0.192	2.472	3.098
2.00	3.34	0.173	2.173	2.602
1.68	3.11	0.150	1.830	2.104
1.41	2.98	0.138	1.548	1.703
1.13	2.64	0.108	1.238	1.298
0.78	2.27	0.080	0.860	0.882
0.40	1.65	0.042	0.442	0.455

* n denotes uniform flow, and p denotes flow at the piezometer section.

Since it is a requisite, for the proper functioning of the control, that the channel be built so that the depth for any flow, at the piezometer section, will be greater than the depth for uniform flow, it is obvious that the slope must be greater than any calculated slope in Table 2. The effect of this will be to cause a backwater curve extending up stream from the piezometer section to the section of uniform flow.

The writer has assumed a slope of 0.0012, with n equal to 0.014, for the purposes of illustration. Table 3 gives the hydraulic elements as computed with these constants for the flows in Table 2.

For the larger flows there will be a considerable drop in the water surface in passing up stream from the piezometer section. The maximum flow that can pass the control section utilizes only a portion of the channel capacity after uniform flow is attained. The channel, as designed on this slope, would carry a maximum flow of 43.5 cu ft per sec, in uniform flow with a depth of 3 ft. The efficiency of the channel, therefore, is only $\frac{28.3}{43.5}$, or 65 per cent. It is the writer's contention that the construction of a channel that can only be utilized for a portion of its design capacity, due to the existence of a bottleneck at the point of flow measurement, is not economical.

Inasmuch as such devices as the Parshall flume[7] and the control meter developed by Julian Hinds,[8] M. Am. Soc. C. E., will accomplish the same purpose with little interference, if any, with the flow of water in the channel above the control, the writer favors their use, especially so because of the ease and accuracy with which flow conditions in the main channel may be predicted.

The writer is of the opinion that the convenience of a straight-line relationship between depth and discharge is not sufficient to justify the uneconomic over-design of the channel section which is necessitated by its use.

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[7] "The Improved Venturi Flume," by R. L. Parshall, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 841.

[8] *Loc. cit.*, p. 859.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

TENSION TESTS OF LARGE RIVETED JOINTS

Discussion

BY MESSRS. CHARLES F. GOODRICH, FREDERICK P. SHEARWOOD, AND JONATHAN JONES

CHARLES F. GOODRICH,[13] M. AM. SOC. C. E. (by letter).[13a]—It is the writer's opinion that the two groups of tests on structural riveted joints, made in connection with the design and construction of the San Francisco-Oakland Bay Bridge, are unequaled in their contribution to the knowledge of the behavior of such joints. Most of those who studied Professor Wilson's paper on "Fatigue Tests of Riveted Joints"[8] were considerably disturbed by the rather low fatigue values obtained, especially for the higher strength steels; but they were more especially disturbed by the low unit stresses at which slip occurred in the high-strength, or the so-called manganese, rivets. It seems to the writer that, in so far as static loads are concerned, the results of the present tests confirm, rather than disturb, one's confidence in riveted joints.

As the factor of fatigue is absent from the tests described in this paper, its authors quite properly have based their conclusions on the static tests of large, full-size specimens. This discusser would emphasize that there is considerable danger in projecting laboratory results on small specimens into the design of structures without a sufficient study of other mitigating factors, such as frequency of maximum loads, yielding of joints and other members and recovery between loads. Literature on projecting laboratory studies on fatigue into actual structures is sadly lacking. The profession needs such interpretation.

The authors have thrown considerable light on the action of riveted joints by considering four stages of joint action: Stage I, in which static friction prevents slip, the end of this stage being at the point at which slip at the middle of the joint takes place; Stage II, in which slip occurs until the rivets come into bearing; Stage III, in which rivets and plates deform elastically; and Stage IV,

NOTE.—This paper by Raymond E. Davis, and Glenn B. Woodruff, Members, Am. Soc. C. E., and Harmer E. Davis, Assoc. M. Am. Soc. C. E., was published in May, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[13] Chf. Engr., Am. Bridge Co., Pittsburgh, Pa.

[13a] Received by the Secretary July 6, 1939.

[8] "Fatigue Tests of Riveted Joints," by Wilbur M. Wilson and Frank P. Thomas, Eng. Experiment Station, *Bulletin No. 302*, Univ. of Illinois, 1938; see also "Fatigue Tests of Riveted Joints," by Wilbur M. Wilson, *Civil Engineering*, Vol. 8, August, 1938, pp. 513–516.

in which yielding of the plates, rivets, or both, occurs until either plate fracture or complete shearing of the rivets results.

Comparing joints of carbon rivets with those of manganese rivets, in which the number of rivets is in proportion to their shearing values, the authors, confirming Professor Wilson, find the average rivet stress at the end of Stage I much lower for manganese rivets than for carbon rivets (roughly, one-half) but, as the stages progress, the manganese rivets come into bearing and "catch up." At the beginning of Stage III they are roughly equal, and in Stage IV the manganese rivets have "come into their own" and their value, as compared to the value of carbon rivets, is about proportional to the strengths of the materials.

Regarding this matter of slip, it seems evident that engineers can no longer consider that the friction between the plates of a joint, due to tension developed by the rivets, is a measure of the joint value, even with carbon rivets in carbon plates—especially as working stresses are being continually increased as structural steel is improved in quality and strength.

With respect to high-strength steels and high-strength rivets, these tests have proved that friction between the plates disappears at stresses at or below the working stresses which are reasonable with such steels, and slip occurs in the connections of a member probably either before, or as soon as it receives its full-load stresses. As the rivets come into bearing at these low stresses, it would seem advisable to have the rivets fill the holes as nearly as possible. The writer believes that the optimum in this respect, with due regard to fabrication costs, has about been reached. Clearance between rivet and rivet hole cannot be reduced without causing expensive difficulty and delays in entering. Furthermore, the limit has about been reached in human strength to handle heavier, higher-pressure, rivet guns. The writer believes that the driving technique now being used on large structures by reputable contractors, under competent inspection, secures results that are reasonably satisfactory. Special endeavor to upset the rivets to fill the holes more completely was made in the case of the San Francisco-Oakland Bay Bridge, including investigation of sample drivings, but it is doubtful if it resulted in much better filling of holes in the actual structure than is accomplished in the usual best practice.

There is a balance in economy that must not be ignored. If structural engineers continue to increase unit stresses and then begin to worry about rivets not entirely filling the holes and several other points of workmanship, they may well question whether economy has really been effected. The rivets in the joints of the tension tests on large riveted joints, presented in this paper, were driven under usual shop practice. Some of the rivets were quite long and it is doubtful whether they filled the holes any better than usual; and yet, the joints all showed consistently excellent results. In fact, they were so consistently good as to inspire the writer with fresh confidence in driving technique and in riveted joints generally, especially large butt or shingle joints, data on the strength and behavior of which have been so sadly lacking until now. These tests confirm, in large joints (as have all similar previous tests on smaller specimens), the peculiar capacity of structural steel to transmit stress efficiently and safely from one part to another by means of rivets in shear and bearing.

The tests are a distinct warning that the connections of members subject

to reversal of stress must be over-riveted, especially when using high-strength rivets. There are not many members subject to reversal, however, in which stresses are large enough to demand high-strength rivets. Continuous structures are likely to have several members subject to reversal, either at the design loads, or at a future over-load. Such members should be connected with an ample number of rivets to prevent slip—perhaps more than have heretofore been used.

The authors have arrived at some rather startling conclusions, one of which has been suspected by some engineers—namely, that designers have been wasting effort trying to maintain an optimum net section at the ends of connections or splices by the use of considerably fewer rivets in the end rows. The tests demonstrate that this is a source of weakness, in that "wide spacing of the end-row rivets (near the edges of the plate) is especially undesirable; * * * differential lateral contraction of the opposing plates at each end of the joint subjected such rivets to lateral strain which, combined with the longitudinal strain, caused premature failures of these rivets." Furthermore, the authors state: "The experimental results bore no consistent relation to the specification requirements for determining net section," and that "It is apparent that there is nothing to be gained by using an elaborate formula to evaluate the effect of pitch on net section for joints of this type." The authors have gone so far in this matter as to offer the proposal that "the allowable loads on riveted tension members be based on, and expressed in terms of, stress in the gross section"; then they propose that the members be so detailed "that the net section be not less than 75% of the gross section."

Based on their considerations of the tests, the authors have offered several recommendations, applicable to design, in addition to the one already mentioned regarding net *versus* gross section. It seems to the writer that these tests and recommendations should be given serious consideration by all structural engineers. There is room in this discussion for only a few points.

The authors have tabulated their recommended working stresses for use in the design of riveted joints in bridges, for steels commonly specified (see Table 21). The working stresses for carbon, silicon, and nickel steels are given respectively as 16.0, 20.0, and 22.5 kips per sq in. on the gross section. Assuming that the net section is the minimum recommended by the authors (namely 75% of the gross), these working stresses give unit stresses on the net section of 21.3, 26.7, and 30.0 kips per sq in., respectively. These unit stresses are quite high for general use in bridge design and are scarcely to be recommended for railroad bridges, light highway bridges, or most heavy highway bridges; and they are, perhaps, applicable only to bridges in which the dead load constitutes a large proportion of the total load—say 50 per cent. They are less, however, than was specified for the San Francisco-Oakland Bay Bridge.

The writer questions whether the authors have not based their recommendation of gross section unit stresses for tension members and 75% ratio on their large tension tests only, which are made up of plates alone. Have they also considered the effect of this recommendation on members built of rolled sections or a combination of rolled sections and plates? Consider, for instance, the simple case of a single angle. Any angle that has one hole for a $\frac{7}{8}$-in. rivet will

have left a net section considerably greater than 75% of the gross. Therefore, a unit stress that is based on balancing the 75% ratio is likely to waste material in rolled shapes and even in plates. The gross-section-unit-stress method would be very easy to use, of course, but it would either call for too tight a ratio, gross to net, or it would make it all too easy to waste material and do away with all ingenuity on the part of designers and detailers directed toward maintaining as large a percentage, net to gross, as possible. It scarcely seems possible to find a unit stress based on gross section that will fit various conditions and not waste material.

The authors recommend a somewhat smaller ratio of increase in working stresses in the high-strength steels, as compared with those for carbon steels, than the ratios of minimum yields or tensile strengths of the mill coupons. Ratios of working stress of 1.25 and 1.40 are assigned for silicon and nickel steels, respectively, and 1.33 for manganese rivets, as compared with 1.00 for carbon steel. The authors' conservatism does not seem to be borne out by the results of the tests but is based rather on theoretical considerations as, when high-strength steels are used, some desirable property, such as ductility, plastic flow, and opportunity for stress distribution, must be sacrificed. The tests seem to confirm, rather than upset, the established custom of basing the unit-stress ratios on the yield ratios. Although fatigue may be a reason for a decreased ratio, such a reason must be kept separate from deductions resulting from the tests under discussion.

There is a factor (which may be called a safety factor) that engineers are prone to lose sight of—namely, that the steels, as produced, have actually better physical properties than the specified minima, because the mills, in order to avoid rejections, "play safe" and aim to stay well above the required minima. The average minima run, as the authors state, from 5% to 10% above the required. This tendency to "play safe" is greater in the production of high-strength steels than the ordinary carbon steels because of the greater monetary loss of rejections.

It seems to the writer that both Professor Wilson's tests[8] and those described in this paper have proved conclusively that rivets are not twice as strong in double shear as in single shear, and that specifications should be written to agree with what seems to be a well-established fact. The authors have recommended a ratio of 1.8.

The tests appear to justify Recommendation (5), which states that the practice of assuming equal shear per rivet, regardless of length of joint, is satisfactory. This is a very reassuring conclusion and inspires further confidence in large riveted joints.

The writer cannot agree with Recommendation (6), which states that, except in comparatively heavy structures where reduction in size or weight of splice is important, there is little reason for using manganese-steel rivets rather than carbon-steel rivets. The authors seem to connect this recommendation to the weight of splice material only, and to forget the economies to be obtained by using manganese rivets, not only in splices, but in the connections of members to gussets, where considerable economy is possible due to saving of material and number of rivets. Furthermore, the use of manganese rivets

makes for shorter connections which are recommended by the authors. There are genuine difficulties involved in detailing connections for high-strength steels without a corresponding high-strength rivet. The writer believes that there is a need for high-strength rivets, although perhaps not exactly of the same chemical constituents as were used for the rivets for the San Francisco-Oakland Bay Bridge and in the tests presented in this paper. A somewhat more usable, perhaps softer, rivet will probably be developed for main members, such as will match silicon steel rather than nickel, as it seems likely that nickel-steel will be rarely used as compared with silicon, which bids fair to continue in wide use. A rivet that can be used with a steel of 45 to 50 kips per sq in. would seem to be the most desirable.

The writer wishes to congratulate the authors on the completion of an important series of tests, and on their careful and scientific analysis of them, resulting in material addition to the knowledge of the behavior of structural riveted joints, and in data which should (and undoubtedly will) be used widely by structural engineers in the preparation of specifications.

Frederick P. Shearwood,[14] M. Am. Soc. C. E. (by letter).[14a]—The distribution of the loads in the rivets and the stresses in the material of large riveted joints is one of the problems that, on account of its complexity and the many indeterminate factors which enter into its resistance, have been avoided by mathematical theorists; and these joints have been designed by simple empirical rules and the designer's common sense or judgment. The adequacy of riveted joints proportioned on the assumption that all the rivets in a joint are equally stressed is a confirmation of the "limit design theory" as presented by J. A. Van den Broek,[15] M. Am. Soc. C. E. There can be little doubt, and this paper confirms it, that it is the ductility of the material which enables the resistance to be safely distributed before the end rivets lose their resisting value.

Although careful designers have had to adopt the practice of disregarding the uneven distortion of material in a riveted joint, they have realized that this is not a really satisfactory method and these tests, and the able presentation of them, are most timely and useful contributions to structural engineering. Many of the findings should be applied to modify standard specifications and practice and so utilize more efficiently the resistance of these joints.

To compute, accurately, the stresses in each rivet of a joint it would be necessary to know the exact clearance in the holes, the exact frictional resistance between all the surfaces in contact, and the relative deformations due to shear bearing and extension of the plates. These and some other factors are rather indeterminate and make the problem unsolvable; but, to visualize the approximate progressive action as a joint is loaded, is very desirable.

The action of riveted joints as described in the beginning of the paper does not appear to be possible if there is clearance in the holes and, unless the frictional resistance is elastic. Considering that there is clearance and that the friction is not elastic, all the stress must be resisted by the first row until it

[14] Cons. Engr., Dominion Bridge Co., Ltd., Montreal, Que., Canada.

[14a] Received by the Secretary July 12, 1939.

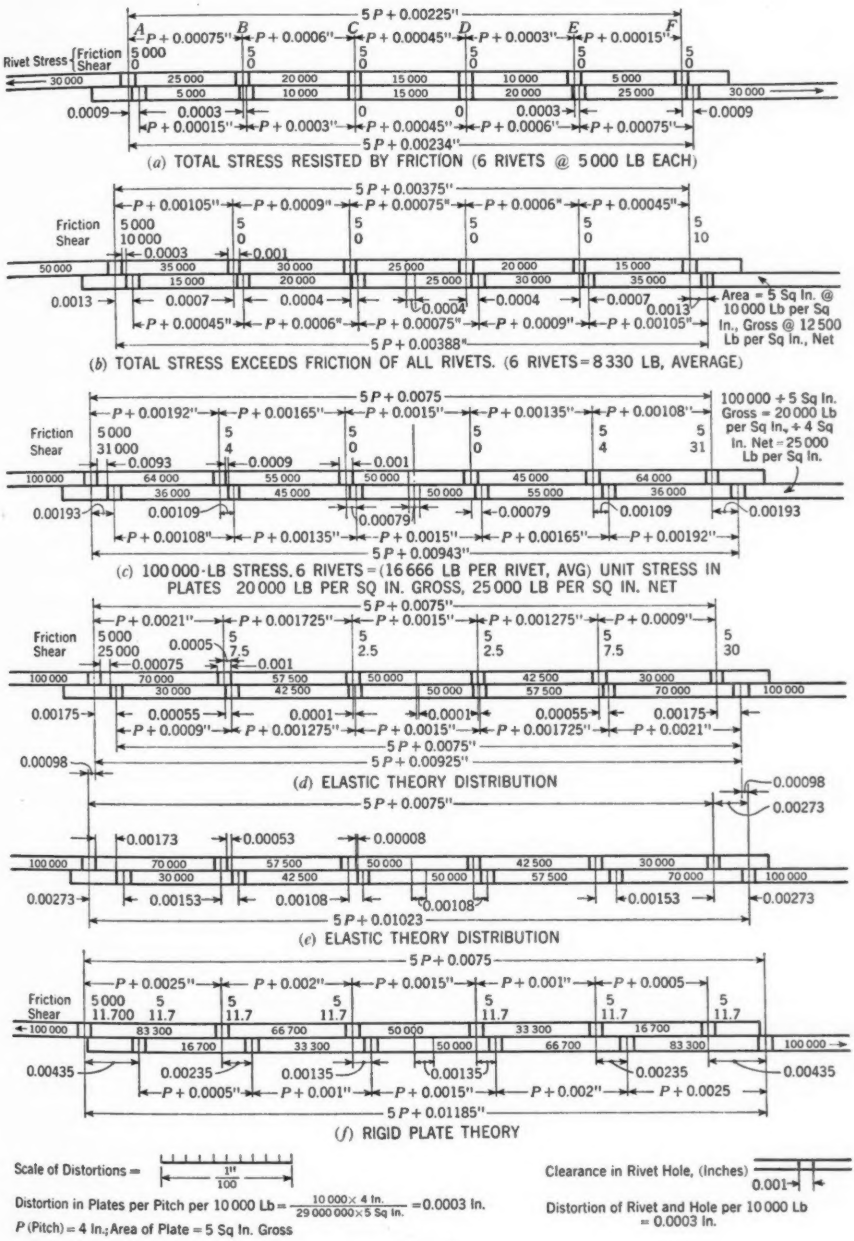
[15] "Theory of Limit Design," by J. A. Van den Broek, *Proceedings*, Am. Soc. C. E., February, 1939, p. 193.

slips, because the strains in the connected plates must be alike if no movement occurs between them. For instance, in Fig. 19(*a*), if no movement occurs the strain in the two plates between Rivets *A* and *F* must be equal, and therefore Rivets *B C D* and *F* are redundant until the plates slip at *A* and *F* when *B* and *E* will resist the surplus stress; and the strain in the plates between *B* and *E* will be equal until they slip and *C* and *D* come into action. The condition when the load is just below the total frictional resistance of all the rivets will be as shown in Fig. 19(*a*) where Rivets *A* and *F* have slipped an amount equal to the difference in the strains in the upper and lower plates from *A* to *F*; whereas the slip at Rivets *B* and *E* is equal to the difference in strains from *B* to *E*. If the load is now increased so as to overcome the friction of all the rivets, the plates will slip until the end rivets come to a bearing and shear in them resists further movement. This condition is shown in Fig. 19(*b*), which shows that the interior rivets still have considerable clearance and so are only resisting by friction. If the load is doubled (see Fig. 19(*c*)), there will still remain a small clearance at Rivets *C* and *D*, and most of the extra load is taken by the first rows, *A* and *F*, and only a small amount by *B* and *E*.

Figs. 19(*d*) and 19(*e*) show the relative position of the plates when the rivets are stressed according to Fig. 1. In Fig. 19(*d*) the general slip is made so that Rivets *A* and *F* are strained to the correct amount to resist 25 000 lb in shear, and it is seen that the other rivets still have clearance and, therefore, should not resist except by friction. In Fig. 19(*e*) the general slip is made so that Rivets *C* and *D* are strained to resist the 2 500 lb in shear, when it becomes evident that the other rivets are strained far above the amounts given by this theory. If the clearances in the holes are eliminated and the friction is elastic, the loads on each rivet will agree closely with this elastic theory; but in this case, as there is no clearance, there should not be any early sudden slip as shown by the tests. Fig. 19(*f*) is an illustration of the rigid plate theory (general practice) which shows that the distortions at the holes are altogether at variance to the assumed shears.

The unit distortions, frictional resistances, and clearances used in making these diagrams are only approximations; but the diagrams may help to visualize what takes place in these joints, and may assist in interpreting the tests.

The tests show that considerable deformation can occur in the rivets and holes at the ends of these joints, and that it is enough to bring the interior rows into considerable effective resistance before the strength of the outer rivets is destroyed. The total slip at the end rivets, according to these tests, may amount to as much as $\frac{3}{8}$ in. before failure occurs. This is far more than the clearance in the holes can possibly be, and must be accounted for by considerable distortion from bearing strains in the rivets and plates, shearing and bending in the rivets, and bending in the plates around the end rivets. It seems likely that the fracturing of the plates through the end rivet holes may be due primarily to initial failure around these holes rather than to the uniformly high stress in the net area of the entire plate. This may indicate why it is better to increase the number of rivets in the end row, thereby reducing the strains around these holes, than to decrease the average unit stress in plates by decreasing the number of holes in the end section.



(a) TOTAL STRESS RESISTED BY FRICTION (6 RIVETS @ 5 000 LB EACH)
(b) TOTAL STRESS EXCEEDS FRICTION OF ALL RIVETS. (6 RIVETS = 8 330 LB, AVERAGE)
(c) 100 000-LB STRESS. 6 RIVETS = (16 666 LB PER RIVET, AVG) UNIT STRESS IN PLATES 20 000 LB PER SQ IN. GROSS, 25 000 LB PER SQ IN. NET
(d) ELASTIC THEORY DISTRIBUTION
(e) ELASTIC THEORY DISTRIBUTION
(f) RIGID PLATE THEORY
Scale of Distortions = 1''/100
Clearance in Rivet Hole, (Inches) 0.001
Distortion in Plates per Pitch per 10 000 Lb = 10 000 × 4 In. / 29 000 000 × 5 Sq In. = 0.0003 In.
P (Pitch) = 4 In.; Area of Plate = 5 Sq In. Gross
Distortion of Rivet and Hole per 10 000 Lb = 0.0003 In.

FIG. 19

The tests were all made with continuous loading and the effect of intermittent loading and stress reversals have not been considered. It seems logical and probable that much of the slip allowed by the clearance in the holes and due to overcoming the frictional resistance, would be recovered on releasing or reversing the load, if the stresses in the plates between the rivets which have slipped and the addition of the reverse load exceeds the frictional resistance of the rivets. Therefore, when reversals must be resisted, the least of the alternative stresses in a joint (with clearance in the holes) should not greatly exceed the frictional value of the first row of rivets if deterioration of the joints is to be avoided.

Clearance in the holes has often been considered as a means of relieving the concentration of stress in the end rivets; but these tests and consideration of the action in them appear to show the reverse, and unless a great proportion of the rivet resistance is due to friction, a better distribution is obtained without clearance. Cold-driven rivets may be the best means of obtaining this result as very little cooling occurs after the driving pressure is removed. Experience with cold riveting in tank work would seem to prove that such rivets do fill the holes thoroughly.

The authors' conclusions emphasize, indirectly, the importance of accurate workmanship because such faults as poor matching of holes must increase the amount of slip, and overstrain the vulnerable rivets. Unstraightened material, etc., must decrease the contacting surfaces and so decrease the friction between the plates. This feature suggests that drilling should give some advantage over punching, as the latter must deform the plates to some extent and so decrease the area of contact between surfaces.

Recommendations (1) and (2) of the paper are of great importance. For some time, there has been a fairly general feeling among designers that the spreading of rivets to save section area was more of a competitive effort to save metal and meet specifications than to gain genuine strength. These tests appear to show quite definitely that nothing is gained by doing it. Recommendation (5) seems rather too broad as these tests and theoretical considerations indicate that the strain in the end rivets must be increased as the length of the joint is increased. It also conflicts with reasons implied by Recommendation (3).

The magnitude and inequality of the distortions in riveted joints, as found by these tests, again emphasize that much of the designing cannot be based entirely on the elastic theories, and they also indicate how useful the ductility of steel can be in offsetting the errors in the assumptions used in designing and in safeguarding the mistakes and abuses of workmanship. The ductile and the excess elastic properties of steel must often function, but seldom are acknowledged.

JONATHAN JONES,[16] M. AM. SOC. C. E. (by letter).[16a]—The engineers of the San Francisco-Oakland Bay Bridge performed a notable service when they appropriated the sum required for a testing program on this large scale. Very

[16] Chf. Engr., Fabricated Steel Construction, Bethlehem Steel Co., Bethlehem, Pa.

[16a] Received by the Secretary August 3, 1939.

seldom can a fraction of such a sum be found for structural research. One of the probable results of this report, however, is that it will justify reliance in future (as other phases of the riveted joint are investigated) upon smaller specimens and less expensive programs. That is, in future it may be adequate to reason from small to large, now that data have been presented on a project that covered both.

In accordance with the desires of those who formulated this program, considerable effort was made to acquire a knowledge of the details of the behavior of these joints prior to their failure. This behavior has been explained clearly in general terms, and for these specific joints certain intermediate loads, called "useful limit point" and "effective rivet yield," have been defined and recorded. This represents an inquiry as to whether such intermediate, rather than the ultimate, loads shall be the basis for determining design loads, by analogy with the customary use of tensile yield point, rather than ultimate strength, of main material.

The answer appears to be in the negative; the record suggests that the intermediate points in question are not so sharply defined, and not so important to structural behavior, but that the ultimate capacity of a joint, much more readily and certainly ascertainable, will suffice for designing.

Examining the lap joints (Table 5) which failed in the plates, one finds that the yield ratio (Column (14) divided by Column (18)) is 0.61 and 0.54 for $\frac{3}{8}$-in. and $\frac{5}{8}$-in. plain carbon plates, and 0.75 to 0.62 for plates in joints, as against 0.64 and 0.56 for the mill coupons corresponding respectively thereto. This ratio is expected to be 0.55 to 0.60 in structural steel. Therefore, if the engineer designs for a certain factor of safety against failure of plates in a joint, he will automatically have an adequate factor against joint yield point.

Examining the lap joints (Table 9) which failed in the rivets, one finds that the rivet-yield ratio (Column (19) divided by Column (26)) is 0.57 to 0.71 for carbon rivets (1 in 5 being less than 0.60) and 0.56 to 0.67 for manganese rivets (1 in 9 being less than 0.60). Again, therefore, if the structural engineer designs with respect to breaking loads he will be in a usual relationship with the "effective rivet yield." The same comment holds true for the ratio between the yield load and the ultimate load for the more complex joints of Table 17, in which silicon plates were connected by carbon rivets.

On the basis of this comment, designers may base riveted joint design upon the breaking strengths determined in tests past, present, and future, with no fear of insufficient protection against some intermediate stage of yield. There is an important exception—the effect of pulsating or alternating loads upon joints which may slip readily; but this is a subject for direct testing in fatigue, rather than for reasoning from phenomena observed in static testing.

It is gratifying that these tests should have led to the authors' fifth conclusion that "the practice of assuming equal shear per rivet, regardless of length of joint, is satisfactory." Their record shows that this may be said of all of the rivets in any one joint. It is not so certain that it should be taken to mean that the same unit shear may be assumed for all of the rivets in a very long joint as for all of those in a very short one. Rivet strength in 7-ft joints was 10% less than in 2-ft joints (heading "Conclusions from the Tests: Effect of Length

of Joint"). This is not a great difference, so long as specification values are not based on the results of the long joints. Rather than do that, the writer would urge that they be based on short joints and slightly reduced for larger ones, since there are possibly thousands of 2-ft joints designed and fabricated for every one of 7 ft.

Before any value can be assigned to a rivet, there must be knowledge of the relation between the ultimate shearing strength of a driven rivet, and the tensile strength of the bar from which it was manufactured. Neither this report nor the pilot test report[8] deals satisfactorily with this important ratio, because it would require that an individual heat be followed through from tension testing of the bar to shear testing of a joint, instead of using averages; and it would require starting with an annealed bar for the tensile test because the cooling conditions make "as-rolled" bars from the same heat so variable in tension test.

This has been done and reported elsewhere;[17] and the writer, in consequence thereof, approves the authors' proposed 15 kips per sq in. shear for carbon rivets (if tied to 52 kips per sq in. specified minimum tensile strength) and their 20 kips per sq in. shear for low-alloy rivets (if tied to 68 kips per sq in. specified minimum tensile strength, annealed). These should be safe values for static loads, as in buildings, and for bridge design if tied in to live loading somewhat in excess of that now operating.

Similar shearing tests (not published) have been made on eleven different compositions of low-alloy rivet steel, ranging from 64 to 83 kips per sq in. in annealed tensile strength. It was found that the ratio "shearing strength of driven rivet to annealed tensile strength of bar" was as low as 0.90 and as high as 1.065. The tests indicate that this ratio should be determined for each particular rivet steel proposed for use; with that information at hand, and with the data of the authors' experiments, the breaking strength of any structural joint in which the rivets are the weaker element will be predicted easily. The same is not true of the behavior of the plate material in joints that are adequately riveted or over-riveted.

Most, if not all, of the tension members in riveted bridge trusses, now standing, have been detailed on a certain assumption which these experiments tend decidedly to disprove. That assumption is that the safe strength at any right section is proportional to the net area existing on that section. Accordingly, economy of main material has been realized by so detailing as to start with a reasonable minimum number of rivet holes across any general section through the body of the member (tack rivets, attachments of diaphragms, etc.) and, on entering a splice or connection, to omit more and more holes as stress is passed through rivets into the splice or connection plate, until in the heart of the splice or connection as many holes are omitted from a section as the prescribed minimum spacing of rivets will allow.

In a typical chord of a bridge built in 1939, for instance, the computed net section at the entry into a connection is 85% of the gross, and in the heart of the

[8] "Fatigue Tests of Riveted Joints," by Wilbur M. Wilson, M. Am. Soc. C. E., and Frank P. Thomas, Eng. Experiment Station, *Bulletin No. 302*, Univ. of Illinois, 1938; see also "Fatigue Tests of Riveted Joints," by Wilbur M. Wilson, *Civil Engineering*, Vol. 8, August, 1938, pp. 513–516.

[17] Report on Silico-Manganese Rivet A195–36T to Subcommittee II of Committee A–1, American Society for Testing Materials, by the Section on High-Tensile Rivet Steel, 1938.

connection it is found to be 73%, the difference being accounted for by stress unloaded into the connection plates. The evidence of the authors' tests is that the section of 85% net area is actually no stronger than that of 73% net area, the reason being found in stress paths and strain effects.

It is natural to inquire at once as to what extent existing bridges are deficient in capacity, as compared with the designers' intention, on the evidence of these experiments. To that end the writer has had computed the factors of safety of all of the splices tested in those experiments, dividing their "adjusted" yield loads and "adjusted" breaking loads by the working loads which would be permitted upon them under the present bridge design specifications of the American Railway Engineering Association (A. R. E. A.). By "adjusted" load is meant the actual experimental load, reduced in the ratio of the minimum yield point, or ultimate strength, of steel permitted under A. R. E. A. specifications, to the corresponding coupon value on the material used in these experiments. The resulting factors of safety, therefore, are presumed to represent the least that could occur in practice, and are less than any average values that might be assumed from any average strength of material as supplied.

These "adjusted" factors of safety are shown in Tables 23, 24, and 25, corresponding respectively to Tables 5, 9, and 17. Each line in these tables represents the average of two identical test specimens. The values in Column (5), Table 23, are obtained from Table 1 by multiplying Column (4), Table 1, by Column (14), Table 1. The adjusted yield load (Column (6), Table 23) equals $\frac{\text{Column (5)} \times 33}{\text{Column (4)}}$, to reduce to the value of the weakest permissible material. The adjusted ultimate load (Column (9), Table 23) equals $\frac{\text{Column (8)} \times 60}{\text{Column (7)}}$, to reduce to the value of the weakest permissible material.

TABLE 23.—TESTS ON LAP JOINTS THAT FAILED IN THE PLATES; STUDY BASED ON TABLE 5

Specimen No.	Net area, in square inches*	Allowable load, in kips*	Coupon yield point, in kips per square inch	Specimen Yield Load, in Kips		Coupon ultimate strength, in kips per square inch	Specimen Ultimate Load, in Kips		A.R.E.A. Safety Factor	
				By test	Adjusted		By test	Adjusted	Yield Column (6) / Column (3)	Ultimate Column (9) / Column (3)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
CCC0	7.77	139.8	44.1	295	221	68.3	485	426	1.58	3.05†
CCC2–2	6.08	109.5	43.4	295	224	69.6	394	339	2.05	3.10
CCC2–3	6.54	117.7	44.1	287	215	68.2	389	343	1.83	2.93
CCC2–4	6.76	121.5	43.9	298	224	69.4	401	347	1.84	2.85
CCC2–5	6.92	124.3	43.4	302	229	70.0	419	359	1.84	2.88
CCC2–6	6.92	124.3	43.7	296	224	69.9	418	359	1.80	2.88
CCC5	5.90	106.2	41.0	287	231	67.9	418	370	2.17	3.49
DCC0	13.14	237.0	38.1	458	396	68.1	840	740	1.68	3.13†
DCC2–3	11.00	198.0	38.9	435	369	67.7	697	617	1.86	3.74
DCC2–4	11.76	212.0	38.1	437	379	68.0	700	618	1.79	2.92
DCC2–4	11.76	212.0	39.1	441	374	68.2	713	629	1.77	2.97
DCC2–5	11.64	210.0	39.9	429	355	67.7	691	612	1.69	2.92
DCC2–6	11.59	209.0	38.5	422	363	67.2	677	604	1.74	2.88
DCC5	9.29	168.0	38.0	411	357	67.6	663	590	2.12	3.51

* Column (2), net area by Specifications of American Railway Engineering Association, 1938, Par. 409; Column (3), the same, multiplied by 18 000 lb per sq in.

† No rivets.

Values for allowable load in Column (6), Table 24, are equal to Column (5) × 13.5 for carbon rivets and Column (5) × 18 for manganese rivets. The values in Columns (7) and (10), Table 24, copied from Table 4, are not very

TABLE 24.—TESTS ON LAP JOINTS THAT FAILED IN THE RIVETS; STUDY BASED ON TABLE 9
(C = carbon; S = silicon; Ni = nickel; and M = Manganese)

Specimen No.	Steel		Rivets		Allowable load, in kips	Coupon yield point, in pounds per square inch	Specimen Yield Load, in Kips		Coupon ultimate strength, in pounds per square inch	Specimen Ultimate Load, in Kips		A. R. E. A. Safety Factor	
	Plates	Rivets	Number	Area, in square inches			By test	Adjusted		By test	Adjusted	Yield Column (9) / Column (6)	Ultimate Column (12) / Column (6)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
ACC18	C	C	18	14.14	191	39.6	549	388	57.2	787	716	2.03	3.75
ACC36	C	C	36	28.28	382	39.6	1 030	730	57.2	1 446	1 318	1.92	3.45
ACC54	C	C	54	42.42	573	39.6	1 640	1 160	57.2	2 120	1 930	2.02	3.36
ASC18	S	C	18	14.14	191	39.6	567	401	57.2	775	705	2.10	3.69
ASC36	S	C	36	28.28	382	39.6	1 178	831	57.2	1 525	1 388	2.18	3.65
ASC54	S	C	54	42.42	573	39.6	1 680	1 188	57.2	2 166	1 970	2.08	3.43
ACM12	C	M	12	9.43	170	49.2	557	430	74.7	715	650	2.53	3.82
ACM24	C	M	24	18.85	340	49.2	1 070	828	74.7	1 460	1 326	2.43	3.90
ACM36	C	M	36	28.28	510	49.2	1 595	1 232	74.7	2 017	1 830	2.41	3.59
ASM12	S	M	12	9.43	170	49.2	565	436	74.7	682	620	2.56	3.65
ASM24	S	M	24	18.85	340	49.2	1 195	924	74.7	1 470	1 340	2.72	3.95
ASM36	S	M	36	28.28	510	49.2	1 610	1 242	74.7	2 054	1 870	2.43	3.66
ANM12	Ni	M	12	9.43	170	49.2	606	468	74.7	704	640	2.75	3.76
ANM24	Ni	M	24	18.85	340	49.2	1 280	990	74.7	1 464	1 335	2.92	3.93
ANM36	Ni	M	36	28.28	510	49.2	1 860	1 440	74.7	2 140	1 940	2.82	3.80
BCC-20a	C	C	22	17.28	233	39.6	685	485	57.2	975	886	2.08	3.78
BCC-20b	C	C	22	17.28	233	39.6	685	485	57.2	1 013	922	2.08	3.95
BCC-20c	C	C	22	17.28	233	39.6	650	460	57.2	988	900	1.96	3.85
CCC7	C	C	8	6.28	85	39.6	280	198	57.2	358	326	2.33	3.84
DCC7	C	C	14	11.00	149	39.6	415	294	57.2	623	566	1.97	3.80

TABLE 25.—TESTS ON BUTT AND SHINGLE JOINTS; STUDY BASED ON TABLE 17.

Specimen No.	Net plate area, in square inches*	Rivet shear area, in square inches	Allowable Load in Kips		Coupon Ultimate Strength, in Kips per Square Inch		Ultimate Load in Kips			A.R.E.A. Safety Factor	
			Plate, at 24 kips per square inch*	Rivets, at 13.5 kips per square inch*	Plates (silicon)	Rivets ($\frac{7}{8}$-inch carbon) Table 4	By test	Adjusted to plate	Adjusted to rivets	Referred to plate Column (9) / Column (4)	Referred to rivets Column (10) / Column (5)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
FSCA	12.42	22.8	298	308.5	91.5	58	949	830	850	2.78	2.75
FSCB	24.84	45.6	596	617	91.9	58	1 850	1 610	1 660	2.70	2.69
FSCD	24.83	45.6	596	617	90.4	58	1 799	1 595	1 610	2.68	2.61
FSCC	36.99	68.4	888	925	91.8	58	2 660	2 320	2 390	2.61	2.59
FSCE	36.54	68.4	877	925	92.5	58	2 688	2 325	2 410	2.65	2.61
FSCF	37.00	69.6	888	942	90.3	58	2 652	2 360	2 380	2.66	2.52

* Columns (2), (4), and (5), by Specifications of American Railway Engineering Association, 1938.

satisfactory because they are averages of more than one heat. These values were obtained from 95 "as-rolled" rods for carbon rivets and from annealed rods for manganese rivets. The adjusted yield load (Column (9), Table 24) equals $\frac{\text{Column (8)} \times 28}{39.6}$ for carbon rivets; and $\frac{\text{Column (8)} \times 38}{49.2}$ for manganese rivets; 28 kips per sq in. is the specified minimum yield point under specifications of American Society for Testing Materials (A. S. T. M.) A141–36 and 38 kips per sq in. (annealed) is the specified minimum yield point under A. S. T. M. Specification A195–39T. The adjusted ultimate load (Column (12), Table 24) equals $\frac{\text{Column (11)} \times 52}{57.2}$ for carbon rivets and $\frac{\text{Column (11)} \times 68}{74.7}$ for manganese rivets; 52 kips per sq in. is the specified minimum ultimate strength under A. S. T. M. Specification A141–36 and 68 kips per sq in. (annealed) is the specified minimum ultimate strength under A. S. T. M. Specification A195–39T.

In Table 25, the adjusted ultimate load for the plates (Column (9)) equals $\frac{\text{Column (8)} \times 60}{\text{Column (6)}}$, and for rivets (Column (10)) equals $\frac{\text{Column (8)} \times 52}{\text{Column (7)}}$, 60 kips and 52 kips being the minimum ultimate strengths for plates and rivet bars specified by A. R. E. A. materials specification. The factor of safety at the yield point was not computed because the joint yield point was difficult to relate to the plate and the rivets, respectively.

For lap specimens failing in plates (Tables 5 and 23), the safety factor for adjusted yield load varies from 1.69 to 2.17 and that for adjusted ultimate varies from 2.85 to 3.51. For lap specimens failing in the rivets (Tables 9 and 24), this safety factor for adjusted yield load varies from 1.92 to 2.92 and that for adjusted ultimate varies from 3.36 to 3.95. For butt and shingle joint specimens (Tables 17 and 25), this safety factor for adjusted ultimate varies from 2.61 to 2.79. As against these may be set the minimum factors of safety intended by the specification writers, $\frac{33}{18} = 1.83$ for yield load, and $\frac{60}{18} = 3.33$ for ultimate.

One interesting circumstance is that, as regards "adjusted yield load," the lap specimens CCCO and DCCO, both of them plain material without any holes or rivets, are more deficient in factor of safety than any of the riveted specimens. This is not true as regards ultimate strength. Deficiency in joint yield point, therefore, is not to be taken too seriously as a criticism of rivet arrangement.

The ratio $\frac{1.69}{1.83} = 92.3$ per cent. Therefore, existing margins against yield point, in carbon steel design, are apparently never below the intention by more than 8%, and seldom below at all; but $\frac{2.85}{3.33} = 85.5\%$ and $\frac{2.61}{3.33} = 78.3\%$; and therefore existing margins against ultimate, in carbon steel design, are occasionally as much as 15% below the intention, and in silicon steel they are habitually lower yet. Viewing this latter statement from another angle, it is as if carbon steel members had occasionally been designed for $\frac{18}{85.5} = 21$ kips

per sq in. and silicon steel members for $\frac{24}{78.3} = 30.6$ kips per sq in., neither of which values would meet with general approval in a specification for a new bridge.

The authors wish to place tension-member design on a basis more in accord with their test results, and at the same time they apparently desire not to add too greatly to the current costs of bridgework. Accordingly they recommend: (1) That tension members be designed for gross section; (2) that 25% be taken out of the critical sections and no more be permitted to be taken out at other sections; and (3) that the allowable unit stress on gross section of carbon steel bridge members be 16 kips per sq in.

The writer believes that the first recommendation involves too many difficulties to be acceptable. For instance, many tension members are being built to-day with top and bottom cover plates, pierced with oblong holes, say, 12 in. wide, to permit access. These cover plates are reduced in net width by nearly 50 per cent. Other irregularities are encountered in detailing. Designers will have to treat such cases on the basis of the picture that exists, and the writer feels that a net-section basis, however modified, will necessarily be retained.

Since additions on the order of 10% to 15% to the gross section of riveted tension members are not acceptable economically, designers will begin a search for expedients whereby no such additions need be made if the findings of these tests are accepted. The authors realize this, and they have offered such an expedient, namely (see Recommendation (3)), that a gross unit stress of 16 000 lb per sq in., which is equivalent to 21 333 lb per sq in. on a 75% net section, be substituted for the 18 000 lb per sq in. of net section found in current specifications.

As one argument for such increase, the authors cite the excess of coupon strength they have found over the specified minimum, amounting to 10% at yield point and 5% at ultimate. To the writer this excess seems not available for design purposes. It is true that he has seen few yield-point reports that did not exceed the specified minimum, but he has seen many reports of ultimate tensile stress that were a few hundred pounds under or a few hundred pounds over. To depend, definitely, on all steel being, by some percentage, stronger than the specification requires is illusory. Designers should continue to assume that steel of the minimum permitted strength may be built into any critical part of their structures.

Another argument for increasing allowable unit stress in a joint is found by the authors in the fact that the ratio, $\frac{\text{Breaking load per square inch}}{\text{Coupon strength}}$, is greater than 100% (103.6 and 105.7) for Specimens CCC5 and DCC5 (all holes out), whereas it varies from only 83.3 to 96.7 for the nine Specimens CCC2 and DCC2 (two end holes). These findings are confirmed by E. L. Gayhart[6] in the following words:

> "A plate failure through the net section of full-row chain riveting may be expected to show a strength in excess (possibly 10 percent) of the theoretical value. The omission of alternate rivets in the outer row

[6] "An Investigation of the Behavior and of the Ultimate Strength of Riveted Joints Under Load," by E. L. Gayhart, *Transactions*, Soc. of Naval Architects and Marine Engrs., Vol. 34, 1926, p. 55.

causes a loss of strength in the net section failure as compared with full-row riveting, and may even show less than theoretical strength."

Further confirmation is found in wide-plate tests by Wilson, Mather, and Harris[18] in which it appears that riveted lap joints with rivet spacing the same in all rows showed unit strengths on net section exceeding the coupon strength by 6.5% for two rows and by 7.5% for four rows. Other joints with certain rivets omitted from the outer rows, while their total capacity was thereby increased, showed less unit strength on net section than the coupons.

However, it will be noted that these reported increases of 3.6% to 10% over coupon strength, in specimens with all holes out, do not check with the authors' proposed increase from 18 000 to 21 333, or 18.5%, in working stress.

The increase over coupon unit strength was obtained by the authors (in Specimens CCC5 and DCC5), by Gayhart, and also by Wilson, Mather, and Harris on specimens with closely and uniformly spaced gage lines, creating a very straight and uniform flow of stress. Practically, such a condition cannot exist in built-up members, where flange angles, for instance, prevent this perfectly uniform spacing of gage lines. Before the results of Specimens CCC5 and DCC5 can be taken as a basis for design, they should be checked by similar specimens, except that the spacing of the gage lines should have variations up to at least an inch; and by still other specimens in which most of the 25% area deducted should be collected in one hand hole.

Furthermore, the even spacing of gage lines is not the only factor in favor of Specimens CCC5 and DCC5 in these tests. Comparing their designs (see Fig. 3), the distance from center of splice, where both plates are carrying the same unit stress, to the extreme rivet row where the stress in one plate is doubled and in the other reduced to zero, is seen to be 2 in. for Specimen CCC5 and 6 in. for Specimen CCC2–2. It is 6 in. for Specimen DCC5 and 9 in. for Specimen DCC2–3. Is it possible that the effect of differential plate stretch, elsewhere cited by the authors as a reason for failure in the end row, was here a little favorable to Specimens CCC5 and DCC5? It would seem that further tests are indicated, gradually increasing the pitch between the two rows of Specimen CCC5 and thus increasing this differential stretch.

Furthermore, some tests should be run in double shear. A factor which should thus be eliminated is the influence of bending on lap specimens. Whatever this factor may be, it may have been different on the short joint, CCC5, and the longer joints of the CCC2 series.

Therefore, the entire question of specification revision should await a more complete range of tests, simulating a number of practical arrangements of parts (plate with two flange angles, etc.) in which it is impossible to achieve an ideal stress path. It would seem that lighter and smaller specimens would serve this purpose.

Until further information is thus obtained, it would be premature to adopt a tensile stress of 16 kips per sq in. of gross section simply because of the behavior of the more or less ideal Specimens CCC5 and DCC5.

The more probable, although only implied, basis for the authors' recom-

[18] "Tests of Joints in Wide Plates," by Wilbur M. Wilson, M. Am. Soc. C. E., James Mather, and Charles O. Harris, Eng. Experiment Station, *Bulletin No. 239*, Univ. of Illinois, 1931, Table 9, p. 44.

mendation as to unit stress is that the factors of safety corresponding thereto have been shown (as outlined in the first part of this discussion) to be all that exist in at least some parts of some existing bridges (and can at any time exist, under current specifications); and these bridges appear to be satisfactory. It seems doubtful that this will be a general attitude. It is possible that, in course of time, designers might adopt, and detailers would welcome, the principle of taking all holes out of all rows in splices and connections. It seems less likely that they will pass quickly from 18 to 21 kips per sq in. as a permissible unit stress on such sections.

It may be of interest to cite a test made at Lehigh University, in 1939, by Bruce G. Johnston, Assoc. M. Am. Soc. C. E.[19] A bar 17.5 in. wide was reinforced with a pair of pin plates attached by fillet and plug welding. In front of the pin, the pin plates tapered toward the center of the bar.

The welds at the forward end of the pin plates, where the differential elongation appeared in the authors' tests to be so severe on rivets, showed no distress. These fillets were equal to, but not in excess of, the requirements of the American Welding Society Specifications for Welding of Bridges. After failure (through bolt holes in the body of the bar), the whitewash had not scaled away from these welds.

There is an interesting lead here toward an expedient for joint design to meet the situation revealed by the authors' tests—namely, before entering a riveted joint or splice, to weld on a reinforcing plate of such dimensions that full gross section can be used in the design, and full-row riveting can be used throughout the joint. This expedient must be regarded with caution because (a) its effect upon strength in fatigue remains to be proved by test, and (b) fabricators are not prepared to do welding on main members in riveted construction except at relatively high cost. Nevertheless, for long members and for fairly steady stresses, this expedient appears to the writer more valid than a considerable upward revision in tensile unit stress.

To summarize these comments, the experiments reported in the paper are important to structural designers and demand new thinking with respect to tension-member design. They point, however, toward many additional tests to be made before general changes in specifications are undertaken.

Corrections for *Transactions*: In Table 1, the column headed "Figure number," change "3" to read "2," "6" to read "2, 3 and 4," "7" to read "2" and "14" to read "4"; on page 815, Line 8, change "Table 5" to "Table 9"; on page 815, Line 41, the parenthesis "(see Column (15), Table 9)" should read "(see Tables 5 and 9); on page 828, Line 5 from the bottom, "Table 5" should read "Table 9"; in Table 14 insert new second column headed "Pitch, in inches" with values of 3, 4.5, and 6 on corresponding lines; on page 857, Line 10 from the bottom, "Illinois" should read "California"; in the sub-caption to Fig. 18(*a*) delete "(2 Plates and 76 Rivets)"; in Fig. 12, a corrected figure in *Transactions* will show heavy lines denoted by the legend; and, in Figs. 11(*b*), 11(*c*), 14(*a*), and 14(*b*), the upper curves in each case should be translated slightly to the left to line up strain abscissas with the corresponding rivets of the joint beneath.

[19] *The Welding Journal*, Am. Welding Soc. August, 1939, p. 253–5.

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DISCUSSIONS

HYDROLOGY OF THE GREAT LAKES
A SYMPOSIUM

Discussion

BY MESSRS. ALFRED J. COOPER, JR., AND ADOLPH F. MEYER

ALFRED J. COOPER, JR.,[15] JUN. AM. SOC. C. E. (by letter).[15a]—Mr. Hickman's paper is an interesting treatise on the subject, and it is with some reluctance that an attempt is made to add to, or detract from, the value of his conclusions. However, the impression is given that some of his statements are based either upon conditions which are peculiar to the Great Lakes region where the experiments were conducted, or upon insufficient data of relative humidity. The purpose of this discussion is to show that, in the Tennessee River basin, there is a marked variation of relative humidity with evaporation (or *vice versa*), as well as the variation of evaporation with wind velocity.

The Hydraulic Data Division of the Tennessee Valley Authority has collected daily evaporation data at four stations in the Tennessee River basin from the fall of 1934 to the present date, 1939. These stations contain equipment in accordance with U. S. Weather Bureau Class "A" evaporation station specifications. A brief description of the two stations from which the data used in the discussion were obtained follows:

Norris, Tenn.—The station is situated at an elevation of 1 000 ft above mean sea level, approximately 20 miles, air-line, northwest of Knoxville, Tenn. It is about 4 000 ft south of the Norris Dam and Reservoir but only 500 ft south of the Clinch River. Observations were started on October 23, 1934, and additional instruments, other than standard equipment, were installed subsequently. These additions are: A weighing evaporation pan, two sets of maximum and minimum water thermometers (one set for each pan), a recording hygrothermograph, a water-temperature recorder (in the standard pan), a recording anemometer, and a barograph.

Murphy, N. C.—The station is approximately 70 miles east of Chattanooga, Tenn., in the town of Murphy, N. C., at an elevation of 1 575 ft above mean

NOTE.—This Symposium was published in April, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1939, by A. A. Young, Assoc. M. Am. Soc. C. E.

[15] Junior Hydraulic Engr., TVA, Knoxville, Tenn.

[15a] Received by the Secretary May 25, 1939.

sea level. It is about 700 ft north of the Hiwassee River, and at the extreme upper end of the pool to be formed behind the Hiwassee Dam. Observations were begun on November 17, 1934, and since that date the following instruments were added: A recording hygrothermograph, maximum and minimum water thermometers, and a recording rain gage.

The other two stations are at Beetree Dam, N. C., and Savannah, Tenn. Prior to January, 1939, the latter station was situated at Pickwick Landing Dam, Tenn. Data from these stations are in accord with the trend of Fig. 8(*a*) but are not plotted in detail on that diagram.

To confine the study, the range of air temperature was selected arbitrarily as 40° to 46° F, and that of the water temperature as 44 to 50 degrees. A wide variation in relative humidity over this range was found, and a total of 78 points was used. Data of days on which rain and snow occurred were not used, in order to eliminate any possible errors due to differences of rainfall catch between the evaporation pan and the rain gage.

In the Tennessee Valley experiments, wind velocities are measured by an anemometer beside the pan, with the cups about 1 ft above the water surface, in accordance with U. S. Weather Bureau recommendations. As a result, these velocities are smaller than those at the same instant several feet above the ground. In the particular data used, the maximum average daily ground velocity of the wind was 5.01 miles per hr.

The integrated averages of the daily air temperatures and relative humidities were determined from the hygrothermograph charts. At the Norris station, the integrated daily averages of the water-temperature recorder charts were used, but it was necessary to utilize the daily average of the maximum and minimum water thermometers at the Murphy station. It is believed that no correlation of evaporation and relative humidity is possible when only one daily observation is made of relative humidity unless the relative-humidity curve for the entire day has little or no variation. For any other condition it is necessary to obtain a true daily average by frequent psychrometer or wet-bulb and dry-bulb thermometer readings over 24-hr periods or from a relative-humidity recorder.

Mr. Hickman states (see heading, "Factors Affecting Evaporation"): "The evaporation, obtained as a result of the experiments, cannot be related directly to relative humidity or barometric pressure as measured at standard Weather Bureau Stations," and "No evidence could be found that the amount of evaporation varied with either the relative humidity as read at the station or as determined at neighboring Weather Bureau Stations." The latter statement may be true if it is interpreted exactly as it is written because the average daily relative humidity is obtained only by many readings throughout the 24 hr—not by one or two readings. In the Tennessee River basin it is not uncommon for relative humidities to vary from a minimum of 25% to a maximum of 99% or 100% within 24 hr. With such a variation an observation of relative humidity taken at any time of the day could not represent a 24-hr average, except by chance.

A plot was made of daily evaporation *versus* average daily relative humidity for the data selected. Fig. 8(*a*) shows the points as well as the most likely

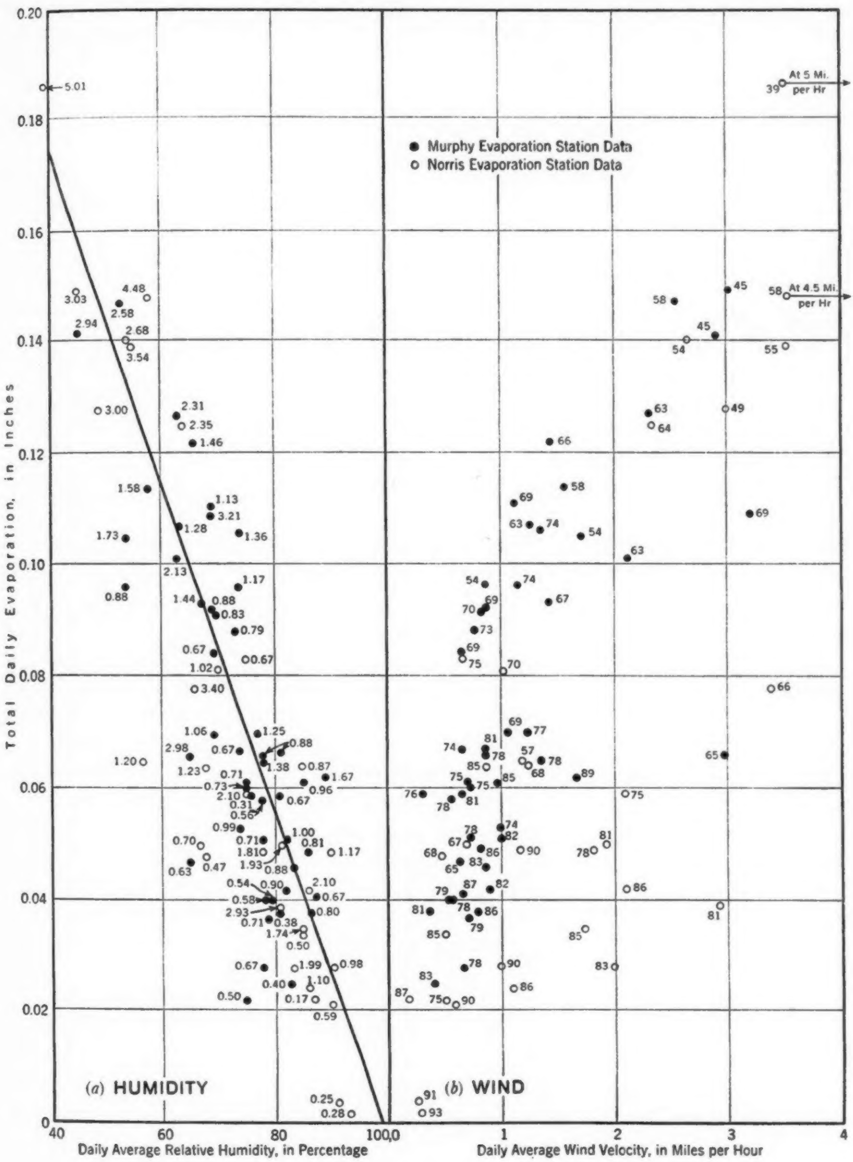

FIG. 8.—VARIATION OF EVAPORATION WITH RELATIVE HUMIDITY AND WIND

straight line to indicate the trend. The average wind velocity, in miles per hour, is noted beside each point. For large rates of evaporation there are correspondingly large wind velocities; but for small rates there is a heterogeneous mixture of large and small velocities, in many of which the large velocities approach or exceed those in the higher evaporation "region" of the diagram. It is apparent from Fig. 8(*a*) that evaporation does depend definitely upon relative humidity. The trend toward decrease in evaporation with increase in humidity is clearly shown by the slope of the trend line. It is not intended that this line be construed as representing the equation of evaporation, but rather that it be considered as showing merely the distinct trend of the plotted points. Were there no connection between evaporation and relative humidity, such a scattering of the points would be anticipated that the eye could not envision any trend. For a given evaporation the maximum variation in humidity from the line is 20%, and the average variation of all the points from the line is only 5 per cent. It must be remembered that the other factors influencing evaporation are concurrently active in these field observations and affect the foregoing relationship.

Another plot was made of daily evaporation *versus* wind velocity for the same observations used in Fig. 8(*a*), and the corresponding relative humidity was noted beside each. The result is the "shot-gun" pattern of Fig. 8(*b*) which, nevertheless, shows certain interesting features. The main conclusion to be drawn from this figure is the dependence of evaporation upon wind velocity. However, the effect of relative humidity is apparent even in this diagram. For evaporation exceeding 0.100 in., 95% of the points plotted show relative humidities less than 70%, and 78% of the relative humidities are lower than 65 per cent. For evaporation less than 0.100 in., 82% of the points show relative humidities above 70%, and only 3% show relative humidities less than 65 per cent. A curve was not fitted to these points, but a trend is indicated toward an increase in evaporation with increased wind velocity. A substantiation of the conclusion drawn from Fig. 8(*a*), that there is a trend toward an increase in evaporation with a decrease in relative humidity, is likewise apparent. Together, Figs. 8(*a*) and 8(*b*) show definitely that evaporation, as measured from pans in the Tennessee River basin, depends more upon relative humidity than upon wind velocity, because the scattering of the points in Fig. 8(*a*) is not as great as in Fig. 8(*b*).

Mr. Hickman refers often to the assumption that the relative humidity corresponded to the average conditions that existed during the particular period of the year in which the data were obtained at the four stations. Data observed in the Tennessee River basin show that there is nothing to indicate that relative humidity remains near a practically constant average for any particular month, as do, for example, air or water temperatures. Table 3 gives the monthly average relative humidity from 1935 through 1938 for the Murphy and Norris stations, respectively, for the period used in this discussion, and the data substantiate this fact.

For the points used in Fig. 8, the daily average relative humidity varied from 39% as a minimum to 93% as a maximum. In this area it cannot be said that there is an "average condition" of relative humidity for a period

TABLE 3.—AVERAGE RELATIVE HUMIDITY

Year	January	February	March	April	October	November	December	January	February	March	April	October	November	December
	(*a*) Murphy Evaporation Station							(*b*) Norris Evaporation Station						
1935	67	65	61	62	72	76	73	72	69	69	66	71	81	80
1936	77	74	68	61	86	79	83	78	74	72	68	90	80	85
1937	83	72	71	73	84	80	88	88	..*	56	56	72	71	78
1938	81	80	77	77	76	79	80	76	77	69	61	64	71	69

* Report not complete.

even as short as a day since such decided variations exist. Table 3 indicates such changes from year to year in monthly average relative humidity that it is difficult to consider an average value to be used with measurements of evaporation in a study of field conditions. One wonders whether relative humidity is not one quantity of the atmosphere of which the seasonal trend from year to year is erratic and unpredictable, in so far as estimating closely what can be expected for a certain month or season. This variation is dependent largely upon the frequency of rainfall, wind conditions, and air temperatures. These factors then influence the rate of evaporation through the relative humidity they produce.

No definite relationship was found from the barograph charts of the Norris station which would show a correlation between evaporation and barometric pressure. However, daily pressure variations at this station are small—rarely more than 0.5 in. of mercury, which is less than 2% of the mean barometer reading. Results of the evaporation experiments at Fort Collins, Colo.,[16] indicate an approximate increase of 2% in evaporation for each inch of decrease in barometric pressure. Since the maximum variation in pressure at the Norris station is only 0.5 in., the maximum variation that could be attributed to barometric pressure change is about 1 per cent. This variation would be so small that it could not be detected in an observation involving such small magnitudes.

In conclusion it may be stated that the evidence presented indicates that evaporation is influenced by both relative humidity and wind velocity, but with the greater effect being due to relative humidity. It is believed that Mr. Hickman is correct in suggesting that current opinion as to the effect of wind on evaporation is in need of revision—but this revision should be toward giving less weight to its effect and more to that of relative humidity.

ADOLPH F. MEYER,[17] M. AM. SOC. C. E. (by letter).[17a]—The subject matter of this Symposium is of recognized importance. There is great need for observations of evaporation from cold water during the summer and from warm water during the winter. The following discussion relates to the

[16] "Evaporation from Free Water Surfaces," by Carl Rohwer, Assoc. M. Am. Soc. C. E., *Technical Bulletin* No. 271, U. S. Department of Agriculture, December, 1931.

[17] Cons. Engr. (Meyer Governor Co.), Minneapolis, Minn.

[17a] Received by the Secretary June 2, 1939.

Duluth evaporation records and to evaporation from Lake Superior, without reference to conditions on the remainder of the Great Lakes.

In his paper, Mr. Hickman states (see "Synopsis") that "an effort was made to keep the temperature of the water in the pans approximately the same as the temperature of the water in the open lakes"; and (see "Description of Equipment: Design of Pan") that "Due to mechanical difficulties, the control was far from perfect; but the results were entirely satisfactory when analyzed by the method which was used and which does not require that the pan water be at the same temperature as the lake water." It may be quite true that the pan water need not be at the same temperature as the lake water at any given time, but the range of evaporation observations should certainly cover the general range of conditions prevailing on Lake Superior.

An examination of the records filed in the Engineering Societies Library[10] reveals the fact that, at the Duluth station, only about 75 days' records are available which are reasonably applicable to fall and winter evaporation from Lake Superior, including the months of September to February, inclusive. The arrangement for heating the evaporation pan during the winter apparently functioned satisfactorily. In general, the results secured indicate high winter evaporation of approximately the same magnitude determined by the late John R. Freeman,[9] Past-President and Hon. M. Am. Soc. C. E., through the use of the evaporation formula (Equation (5)), utilizing the writer's wind factor.[8]

The situation respecting summer evaporation from Lake Superior is quite another matter. No records whatsoever are on file which even reasonably simulate the conditions prevailing on Lake Superior from March to August. At this time, the Lake Superior water is cold and the air is warm.

In Table 4 the lake temperature given in Fig. 5 is compared with the actual recorded temperature of the water in the evaporation pan.

TABLE 4.—COMPARISON OF OPEN-LAKE TEMPERATURES

Month	Mean Water Temperature		Difference, in degrees Fahrenheit	Month	Mean Water Temperature		Difference, in degrees Fahrenheit
	Open Lake	Evaporation Pan			Open Lake	Evaporation Pan	
April	33	41.8	8.8	July	47	68.1	21.1
May	37	53.5	16.5	August	50	69.3	19.3
June	42	62.9	20.9	September	51	57.6	6.6

Observations of pan evaporation, with water temperatures ranging 20° higher in the pan than in the open lake, can scarcely be accepted as a measure of summer evaporation from Lake Superior. This phase of the subject is of the utmost importance, since the main difference between the conclusions of both papers and those of some of the earlier investigators results from differ-

[10] A copy of the thesis ["Evaporation from the Great Lakes"], containing all observational data, has been filed for reference in the Engineering Societies Library, 33 W. 39th Street, New York, N. Y.

[9] "Regulation of the Great Lakes," by J. R. Freeman, 1926 Edition, p. 135.

[8] "Elements of Hydrology," by A. F. Meyer, p. 238.

ences in summer evaporation, and the assumption of summer underground flow and evaporation being somewhere near equal. This also has a bearing on the conclusion that land losses are only 5 in. or 6 in. a year, as against the normally assumed losses of from 14 in. to 22 in., depending upon precipitation and temperature.

In the paragraph preceding Equation (4), Mr. Hickman states, "In order to determine whether discrepancies, due to location or methods, existed in the data, charts for each station were made. No irregularities in the slope or position of the contour lines were evident."

The writer plotted on a large-scale chart, corresponding to an enlargement of Fig. 4(*a*), all the available daily records of evaporation for Duluth when the wind velocity was between 2.5 and 7.5 miles per hr at the pan level; and there was either no precipitation or less than 0.02 in. The average wind velocity for the plotted points was found to be substantially 4 miles per hr.

The writer then drew on this chart the evaporation lines shown on Mr. Hickman's master chart, Fig. 4(*a*). It appears that about one-third of the daily evaporation observations at the Duluth station plot outside of the limits of Mr. Hickman's master chart. No data whatsoever are available for the Duluth station to form the basis for either the zero or the 1-in. line shown in Fig. 4(*a*). Only a few points are available for the 2-in. line.

A study was made of the 4-in.-per-month evaporation line along which the greater number of the daily evaporation points fell. All of the points lying in a belt extending from half-way between the 4-in. and 5-in.-per-month evaporation lines were assumed to control the 4-in. line. The average of these points was found to be 0.123 in., or 3.7 in. of evaporation per month. The points ranged from 0.04 in. on March 31 (representing 1.2 in. monthly evaporation, with a southwest wind of 3.0 miles per hr and 92% relative humidity at the U. S. Weather Bureau Station) to 0.28 in. on May 17 (representing 8.4 in. monthly evaporation, with a northwest wind of 7.0 miles per hr and 49% relative humidity).

The second most erratic points ranged from 0.05 in. on March 15 (representing 1.5 in. monthly evaporation, with a northwest wind of 3.6 miles per hr and 72% relative humidity at the Weather Bureau Station) to 0.22 in. on March 9 (representing 6.6 in. monthly evaporation, with a southeast wind of 3.0 miles per hr and 53% relative humidity at the Weather Bureau Station). Another typical low reading was 0.09 in. on April 14 with a northeast wind at 3.9 miles per hr and 95% relative humidity at the Weather Bureau Station. Surely these records indicate the effect of large differences in relative humidity in so far as they reflect changes in vapor pressure in the air.

The points controlling the upper and the lower portions of the 4-in. line were considered separately. The average of those points controlling the upper half of the curve represented 5.0 in. of evaporation per month, ranging from 2.4 to 8.4 in. per month. The average of the points controlling the lower half of the 4-in. line represented only 3.2 in. per month, varying from 1.2 in. to 5.4 in. per month. The lower half represented conditions prevailing during the part of the year when the relative humidity was high.

A trial line marked "4 in. per month (Meyer)," shown in Fig. 9, was then drawn from the lower end of Mr. Hickman's 5-in.-per-month line to the upper end of his 2-in.-per-month line. The new line intersected Mr. Hickman's 4-in. line practically in the center of the chart. The average of the points lying within a belt of the same width as that previously used on the author's 4-in. line gave an average of 0.138, or 4.1 in. of monthly evaporation. This is a better average than the author's 3.7 in. for the 4-in. line. Furthermore, the points controlling the upper half of the writer's line averaged 4.5 in. per month, and those controlling the lower half of the line averaged 3.9 in.

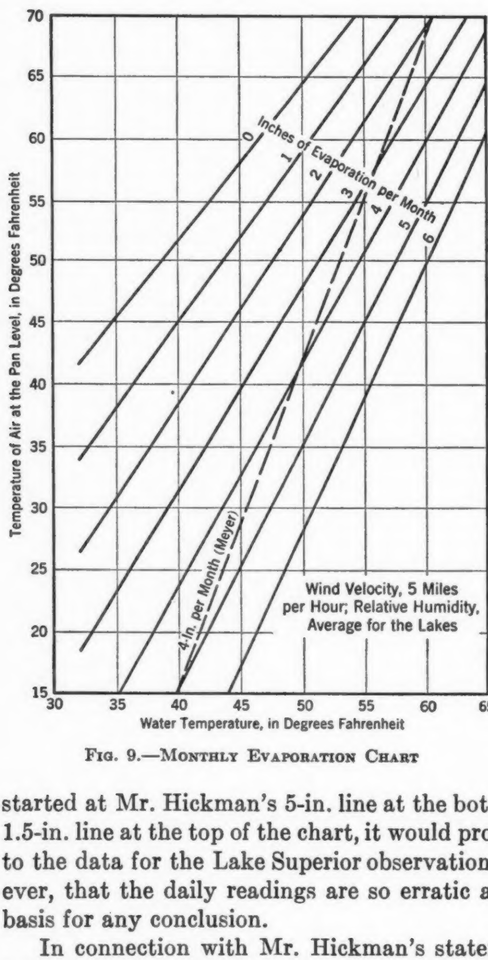

FIG. 9.—MONTHLY EVAPORATION CHART

A second trial 4-in. line was drawn, starting at the same point at the bottom of the chart, but ending at the 1-in. line instead of the 2-in. line. The average of the points controlling the upper part of this line represented 3.5 in. of monthly evaporation, whereas those controlling the lower part represented an average of 3.9 in. of monthly evaporation. Evidently, if the 4-in. line started at Mr. Hickman's 5-in. line at the bottom of the chart and ended at his 1.5-in. line at the top of the chart, it would probably come nearest to conforming to the data for the Lake Superior observation station. The fact remains, however, that the daily readings are so erratic as to constitute an unsatisfactory basis for any conclusion.

In connection with Mr. Hickman's statement that no relation was found between relative humidity and evaporation, the following facts should be borne in mind:

(1) Observations at the Duluth evaporation pan were made only in the morning at about 9:00 o'clock. The data on file do not cover relative humidity and vapor-pressure gradients above Lake Superior.

(2) The relative humidity observations at the evaporation station are at variance with the observations of the U. S. Weather Bureau situated on the hill in Duluth. At first glance, this variation, illustrated by Table 5, appears contrary to what would be expected.

TABLE 5.—MEAN MONTHLY RELATIVE HUMIDITY

Month	U. S. WEATHER BUREAU		Evaporation Station; 9:00 A.M.
	6:30 A.M.	12:30 P.M.	
January	82	73	63
February	85	70	60
March	82	60	58
April	83	72	64

Although a mean monthly relative humidity of 63% at 9:00 A.M. at the evaporation station would appear to conflict with a reading of 73% made at the Weather Bureau Station at noon, when the relative humidity is normally lower, the facts are that the January mean temperature at the evaporation station is 6.4°, whereas, at the Weather Bureau Station, it is only 3.3 degrees. When the actual vapor pressure in the air is computed from the relative humidity and the temperature for January, it appears that the readings at the evaporation station and at the Weather Bureau Station give nearly identical results, and indicate a most interesting relationship worthy of further study.

It is a fact that the reduction in relative humidity during the day often represents little more than a variation in temperature, with the actual vapor pressure remaining the same. In the writer's judgment, temperature and relative humidity, considered together, as they are in the Meyer evaporation formula (Equation (5)), are a much better indication of the actual vapor pressure in the air which controls evaporation than temperature alone.

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DISCUSSIONS

RECONSTRUCTION OF THE WALPOLE-BELLOWS FALLS ARCH BRIDGE

Discussion

By J. B. French, M. Am. Soc. C. E.

J. B. French,[4] M. Am. Soc. C. E. (by letter).[4a]—The work described in this paper shows boldness and resourcefulness in planning and much skill and constant alertness in the supervision of its execution. It does not appear that any traffic, either vehicular or pedestrian, had to be maintained at any stage of the work, and therefore the risk of loss of life was confined to the workmen engaged. Nevertheless, it is obvious that any mistake, either in planning or in execution, might easily have wrecked the damaged structure, with the loss of the lives of the workmen and serious property damage. While the work was in progress, it undoubtedly appeared spectacular to those who stopped to give it intelligent attention, but after it was finished the spectacular quality necessarily disappeared and left no monumental features to attract attention. In act, the best evidence that the work described has been completed effectively will be the failure of present observers to discover that any repairs or "reconstruction" had ever taken place.

Work of this kind, like foundations and structural frames hidden from view by subsequent construction, and all falsework and temporary structures for purely erection purposes constitute a most important part of engineering construction; detailed descriptions of such work, such as are given in this paper, are always valuable contributions to the publications of the Society.

It seems to the writer, however, that the value of the paper would be enhanced if a more detailed statement of the actual cost of the various parts of the work, as executed, could be included. In the "Synopsis," the authors state that "the successful bid" on the "design for repairs" submitted by the "consulting engineers who designed the structure" was $120 000; and under the heading "Conclusion" several general statements are made as to costs, but no clear statement is made of the cost in detail of doing the work as described.

Note.—This paper by Messrs. H. E. Langley and Edward J. Ducey was published in April, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[4] Cons. Engr., New York, N. Y.

[4a] Received by the Secretary June 6, 1939.